t. Special interior ballistic studies.

World War I

Harvey Lincoln Curtis

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C. Snow

VERY COMPLETE
BALLISTIC DATA
ON
A 14-INCH GUN AND
TWO ATTEMPTS AT
ITS ANALYSIS

OBSERVATIONS

MADE AT

DAHLGREN PROVING

GROUND

BY PERSONNEL OF

NATIONAL BUREAU OF STANDARDS

IN 1923

ANALYSES BY C. SNOW

IN 1924 AND 1942

PREFACE

During World War I the Bureau of Standards cooperated with the Navy Department and especially with the Naval Proving Grounds in developing instruments for making interior ballistic measurements on major caliber guns. Some of these instruments were used to make measurements during the firing of the primary batteries of several battleships. Much useful data was obtained in this way, but most of this work was done during the structural firing tests of new battleships when the program was necessarily dictated by the requirements of the structural tests. At the close of the war, the Navy and the Bureau of Standards arranged a more favorable test of instruments.

These instruments had been designed for use in making measurements on a 14-inch gun, and the work and expense of firing such guns was such that proving ground tests using such a gun were not considered during the War. When War I stopped, a 14"-50 caliber gun that needed to be relined was available; large quantities of powder were in stock which could not be kept many years; and suitable naval personnel could be assigned to operate the gun. These conditions favored the cooperation of the Navy. More-

over several of the scientists who had been with the Bureau of Standards during the development of the ballistic instruments had taken jobs with the Navy, and willingly accepted temporary duty with the Bureau. Hence a proving ground test was started on October 26, 1923.

The 95 page report is entitled: "The Correlation of Diverse Ballistic Records". This report gives the times at which the projectile passes certain planes as it moves down the gun barrel, the times of definite amounts of recoil and the pressure as a function of time both on the breech of the gun and in the recoil cylinders. It was expected that with this data the friction between the gun and the projectile could be determined, if only approximately. To make this possible, the differential equation involving motion of the gun, projectile and powder was solved. This solution involved the assumption that the burning powder could, at any instant be treated in the barrel as a gas, with a uniform density over each cross-section and an axial density gradient proportional to the pressure. This treatment was appropriately extended to include the pressure in the enlarged power chamber and in the truncated cone connecting the powder chamber to the barrel. The resulting equations are more

difficult to use than the empirical equations then employed in most ballistic computations, and did not fit the data any better, perhaps because of the unsatisfactory measurements of pressure.

With the outbreak of World War II, the data was again analyzed by Dr. Snow. He prepared a paper entitled: "Numerical Test of the Ballistic Equations of the Dahlgren Firing of 1923", C. Snow (1924). This paper seems worth including in this collection, and a photostat copy will be supplied to both the Army and the Navy for their copies.

In the Bureau of Standards volume there will also be bound two manuscripts which are not elsewhere preserved and which are closely related to the above. The first is a mathematical study, entitled "The Motion of a Rotating Projectile. A Preliminary Study by Prof. H. C. Richards in 1918". It is entirely mathematical. The second is entitled: "Report of Observations made by H. L. Curtis on a trip to France and England during April, May and June 1919". The trip was made at the request of the Navy and only matters connected with ballistics are included.

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1. The Correlation of Diverse Ballistic Records

2. Appendix: Ballistic Formulas and their Derivation: 1924 by C. Snow

3. Numerical Test of the Ballistic Equations of the Dahlgren Firing of 1923 by C. Snow, 1942.

4. The Motion of a Rotating Projectile A preliminary study by H. C. Richards

5. Report on Observations at Proving Grounds in France and England during Apr., May and June, 1919 by H. L. Curtis

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THE CORRELATION OF DIVERSE BALLISTIC RECORDS

A REPORT OF THE EXPERIMENTAL FIRING OF A 14"-50 CALIBER GUN NO. 89-L2 AT NAVAL PROVING GROUND DAHLGREN VIRGINIA

OCTOBER 26-30 1923.

REPORTED TO
BUREAU OF ORDNANCE
NAVY DEPARTMENT
APRIL, 1924
BY
THE NATIONAL BUREAU OF STANDARDS

COPY

PREFACE

The experiment covered by this report was undertaken conjointly by the Bureau of Standards and the Naval Research Laboratory with the cooperation of the Naval Proving Ground at Dahlgren, Virginia. In preparing the report an effort has been made to describe the work and give the results in sufficient detail to be comprehensive without allowing the more important features to be lost in a maze of data. A short list of those members of the three organizations referred to above most closely connected with the several phases of the work follows:

H. L. Curtis	Bureau of Standards, Planning and Direction.
H. H. Moore	Naval Research Laboratory, Development of Apparatus, Operation of Laboratory.
C. Snow	Bureau of Standards, Development of Mathematical Formulas for Analysis.
L. T. E. Thompson	Naval Proving Ground, Dahlgren, Va. Collection of Data, Calibration, Operation of Camera.
A. H. Sellman	Naval Research Laboratory, Recording Apparatus, Photography.
R. A. Webster	Naval Research Laboratory, Installation & Operation of Recoil-meters, Powder Pressure Gages, and Recoil Cylinder Pressure Gage.
W. J. Rooney	Bureau of Standards, Operation of Apparatus for Velocity Measurements. Analysis of Data and Preparation of Report.

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THE CORRELATION OF DIVERSE BALLISTIC RECORDS

REPORT ON EXPERIMENTAL FIRING
OF
14"-50 Cal. Gun
at
THE NAVAL PROVING GROUND
DAHLGREN, VA.
OCT. 26-30, 1923.

I. Introduction

1. Purpose and History

During the past seven years the Bureau of Standards, on behalf of the Bureau of Ordnance, Navy Department, has conducted a series of investigations of the performance of large naval guns both on shipboard and at the Naval Proving Grounds. In the course of these investigations a number of new methods of measurement and types of apparatus have been tried out and developed. The details of the development and the results obtained by the use of the different instruments are contained in reports already submitted covering experimental firings on the Arizona, Mississippi, New Mexico, and Tennessee, and at the Proving Grounds at Indian Head and Dahlgren. Upon the establishment of the Ballistic Section of the Naval Research Laboratory, the apparatus was taken over by that section with the intention that the major part of the experimental ballistic program should henceforth be conducted there. Before completing the transfer, however, it seemed desirable to add to the work already done a more comprehensive experiment covering as many as possible of the ballistic factors involved, for the purpose of correlating the results of the measurement of the different individual factors and comparing the accuracy and applicability of diverse methods

for the determination of a given factor.

The value of such correlation and comparison is twofold. It not only provides the most searching kind of a test of the accuracy of a particular instrument but also establishes the relative value of different methods of measurement with reference to the general problem of gun behavior. Consequently such an experiment might be expected to indicate just what combination of measurements is likely to prove most suitable for a comprehensive definition of ballistic performance and so to point out the line along which the future program should best be directed.

It was for this purpose then that the experiment reported herein was undertaken. Its aim was to secure simultaneous records with all the apparatus which had been sufficiently developed to be reliable in order to make possible a correlation and comparison of the individual records.

2. Scope of Test

The particular factors and methods which were chosen for study were:

1. Velocity
   - (a) Ejection Velocity by Muzzle Fingers
   - (b) External Velocity by Contact Screens
   - (c) External Velocity by Solenoids
   - (d) External Velocity by Camera
   - (e) Internal Velocity or Displacement of Shell in gun by Expansometers

2. Powder Gas Pressure at Breech
   - (a) Time-record by Curtis-Duncan Spring Pressure Gage
   - (b) Maximum record by Crusher gages

3. Recoil
   - (a) Motion of gun in recoil by B.S. Recoilmeter
   - (b) Forces opposing recoil of gun
     Recoil Cylinder Pressure by Spring Pressure Gage
   - (c) Recoil Cylinder Pressure by Engine Indicator

(d) Force of Friction between Gun and Slide by Static Measurement
(e) Force of Springs - Initial Compression and spring constant by static measurement

3. Firing Program.

The experimental firing was conducted jointly by the Bureau of Standards and the Ballistic Section Naval Research laboratory with the cooperation of the personnel of the Proving Grounds at Dahlgren. Seven rounds were fired from 14"/50 cal gun #89-L2, three with service charges, three with reduced charges and one with proof charge. The usual Firing Record covering these rounds will be found in Table 1 page 25.

In carrying out the experiment every effort was made to secure a uniformity of the conditions surrounding each round or failing this to secure such data as was necessary to permit of allowance being made for any variations which existed. For this reason the data collected included a careful survey of all the materiel employed in the test. Everything relating to the condition of the gun was carefully noted. Star gauge records along the bore were made just before firing and the dimensions near the muzzle were checked by special micrometer gages as soon as the test was completed. Minute measurements were made of the dimensions of the projectiles used and the supplementary data incidental to firing were made as accurate as possible. In short every factor which might be expected to affect the results was subjected to careful scrutiny and record.

II. Methods and Apparatus.

1. General

The methods and apparatus used in this experiment were for the most part the same as those employed in previous tests and consequently, with a few exceptions, need no description here. The recording mechanism, the modified type G.E. Oscillograph, is now standard equipment at the Proving Grounds and a complete discussion and description of this apparatus may be found in Part II of this Bureau's "Report on the Experimental Firing of the New Mexico". In the same report are contained discussions of the general method of recording displacement, of the individual types of step-by-step gages such as the recoilmeter and the time-pressure gages, and of the devices used to determine the time of events in the firing cycle.

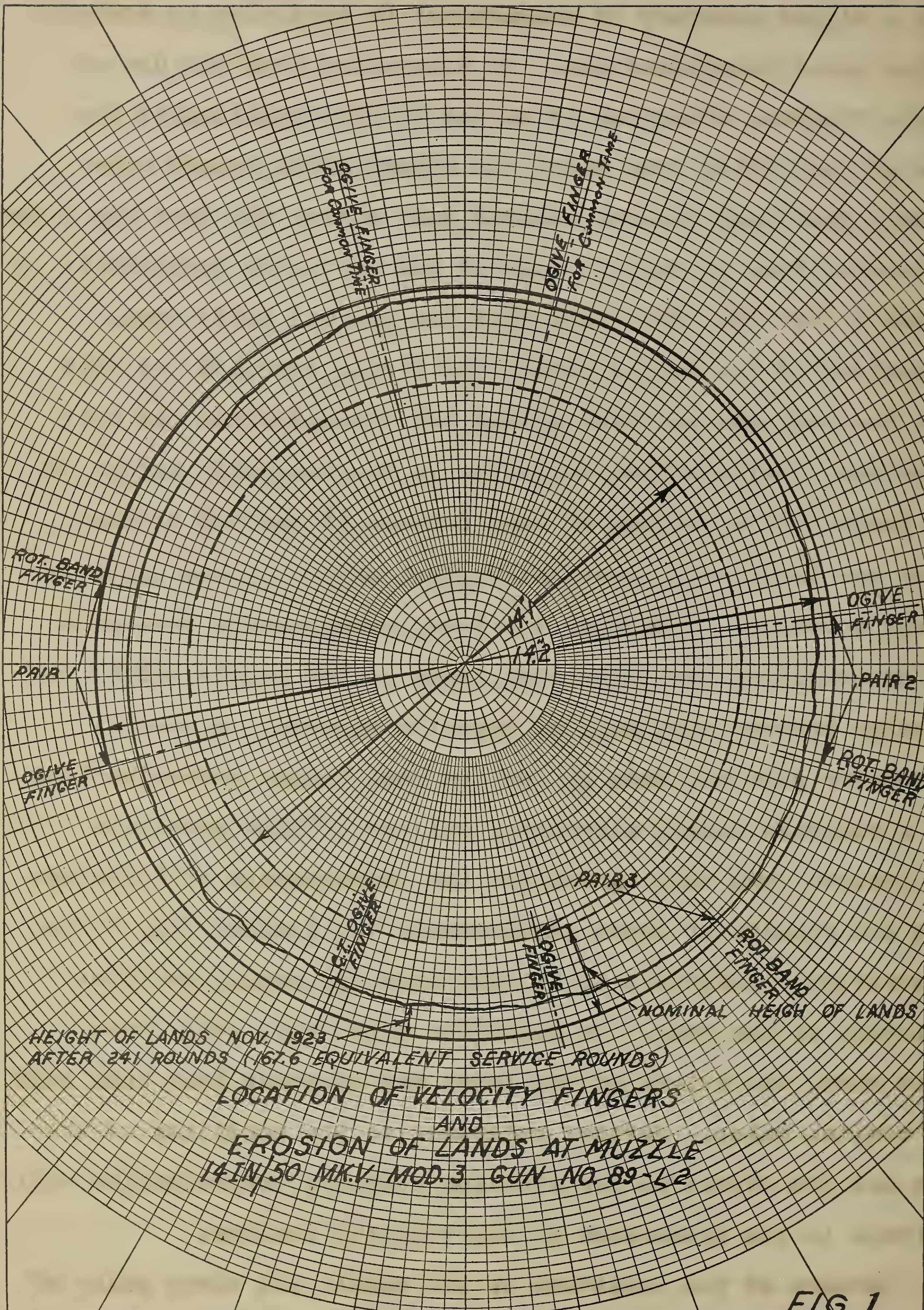
2. Ejection Velocity

The method of determining ejection velocity by the use of muzzle fingers is similar to that used during the calibration firing of the U.S.S. Tennessee and in a number of subsequent firings at Dahlgren during 1922 and 1923. The history of the development of this method together with a description of the type of fingers and blocks finally adopted, are given in a report submitted by this Bureau in March 1923, entitled "Development of Method of Measuring the Velocity of Projectiles Emerging from Gun".

In the present experiment, three pairs of fingers were used in order to check the results of individual pairs and se-

cure a higher accuracy of measurement by obtaining an average of the three determinations. In connection with the use of muzzle fingers, attention is called to the condition of the bore of gun #89L-2 from which the experimental rounds were fired. Star gage records secured just before the test are given in tabular form in Table 3 page 27 and are shown graphically in figure 5 page 30. They show that the gun was badly worn. Inspection of the muzzle before the firing revealed the fact that the wear was not even around the circumference but that there was a well-defined variation in the remaining height of the lands with a maximum of wear between 12 and two o'clock, where the lands were practically gone, and a minimum approximately opposite. Following the test, measurements were made of the height of the individual lands at the muzzle and of the diameter between each pair of opposite grooves. A special micrometer gage was made up for the land height determinations and a standard inside micrometer caliper of high precision was secured from this bureau for the measurements of diameter. From the results of these measurements the diagram Fig. 1 was constructed. Reference to it will show the amount and distribution of the wear. It should be noted that even where least worn, the height of the lands was somewhat less than half its nominal value. Further reference to the condition of the bore and its effect on the ejection velocity determination will be found in the discussion of the velocity results.

Because of the condition of the muzzle the three pairs of fingers were not distributed symmetrically around the muzzle but



LOCATION OF VELOCITY FINGERS
AND
EROSION OF LANDS AT MUZZLE
14 IN/50 MK.V. MOD.3 GUN NO. 89-L2

FIG. 1

were set up in the position shown in Figure 1. The three additional fingers shown on that diagram had nothing to do with the ejection velocity determinations. They made contact on the ogive of the projectile and were used to give a common time record on the individual oscillographs.

3. Contact Screens, Solenoids, and Camera.

Passing to the other velocity determinations, the use of contact screens and solenoids, both recording on the oscillograph, needs no description. The camera and its applicability to the determination of velocity have been described and discussed in two reports submitted by this Bureau, entitled "Work with Projectile Camera at Dahlgren, Va." and dated June 1922, and March 1923, respectively.

4. Displacement of Shell in Gun.

Of the items listed under "velocity", there remains to be considered only the last: vis, the determination of "internal" velocity or the displacement of the projectile inside the gun. The method adopted to determine this displacement is based on the fact that as the projectile proceeds through the bore of the gun, the sudden increase of pressure on a section just passed by it, is accompanied by a radial expansion of the gun at that section. Previous experiments have shown that the expansion so caused is sufficiently great to be detectible and the instrument shown in figure 2 was developed to secure a time record of this expansion at a number of points along the

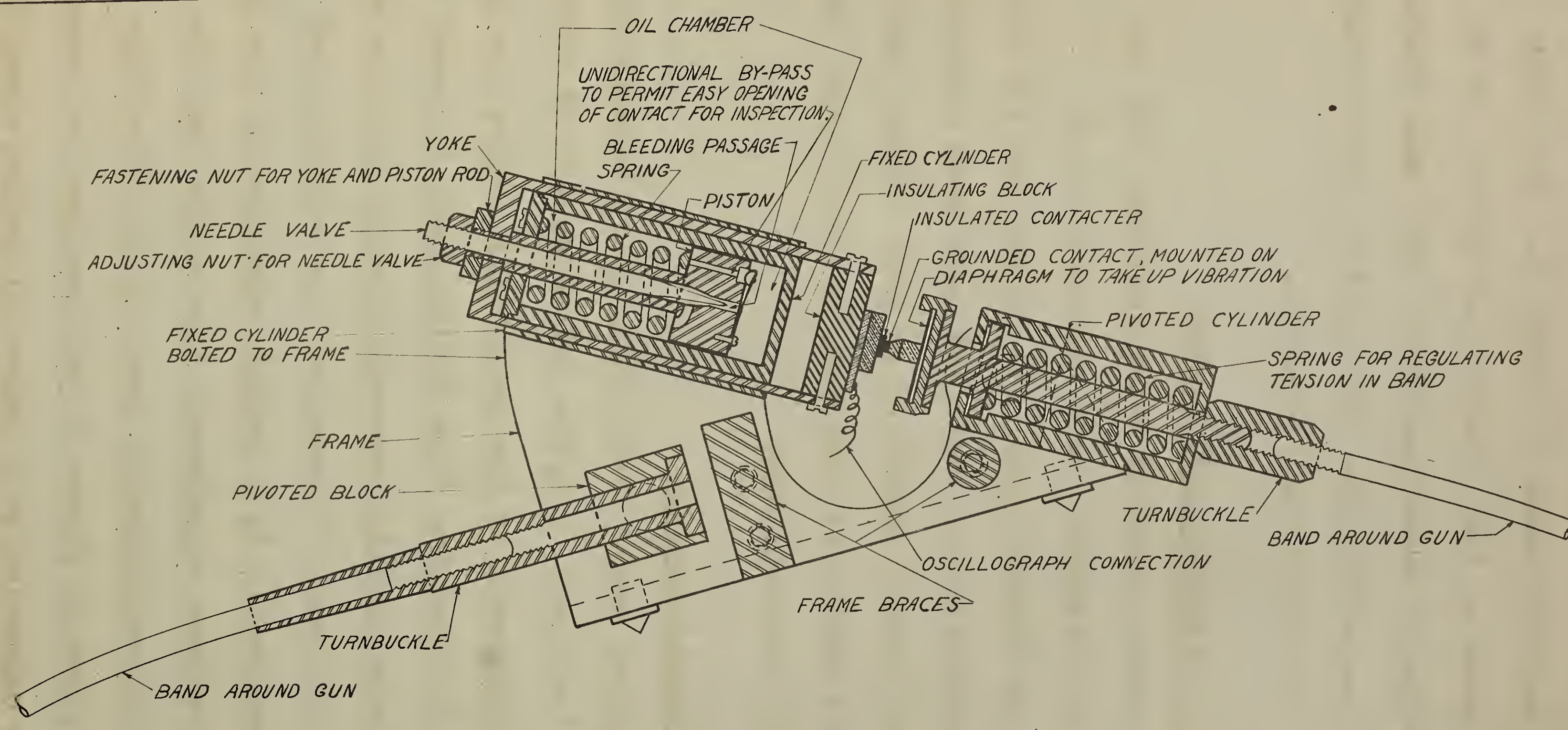


FIG. 2
EXPANSOMETER

EXPANSOMETER
DESCRIPTIVE SECTION
4/1/24
656E

gun. The instrument has been named the "expansometer", although its function is merely to detect the expansion and record the time at which it occurs, rather than, as its name might imply, to measure it.

5. Theory of the Expansometer.

In its essentials, the expansometer may be considered as a stiff split collar or band stretched tightly around the gun with its open ends close together in such a way that any expansion of the gun will cause them to move apart. On one end of the band is mounted an insulated contacter connected in series to a battery, an element of the oscillograph and thence to ground. The other end carries a contact point grounded directly through the band itself and the gun. As initially adjusted the contacter and contact point are just touching so that current flows through the oscillograph circuit. As soon as the gun expands, however, the resulting separation of the ends of the band breaks the circuit and the oscillograph current fails to zero. Hence the time at which the expansion occurs may be determined from an inspection of the oscillograph film on which the usual timing lines are also photographed.

In order to secure satisfactory operation of the device, a number of refinements to the simple scheme outlined in the preceding paragraph were found necessary. Since the expansion to be detected is comparatively small it is necessary that the adjustment of the contact and contacter be delicate in order to insure its opening. And since there must be no possibility of chattering

or alternate breaking and making of the circuit due to mechanical vibration, which might obliterate and would surely confuse the record on the film, it must, at the same time, be rugged and positive. It is moreover necessary that the adjustment be to a certain degree at least, automatic in order that the adjustment be not subject to change due to any slow change in the radius of the gun such as would be caused by expansion due to the gradual heating up of the gun during repeated firing.

In addition to these fundamental requirements, it was considered desirable that the expansometer should require a minimum of attention so that it might be used when firing rapidly or under conditions where the instruments were inaccessible. It was consequently designed to be self closing. That is to say, the instrument was constructed so that the contact points would come together without any manipulation by an operator after being opened by the expansion of the gun.

6. Description of the Expansometer Used.

Having reviewed thus in a general way the aims of the designer, it is believed that the features of the construction of the expansometer as shown in figure 2 may be more readily understood and appreciated. The processes of installation and operation are briefly described as follows. The band or collar around the gun consisted of a suitable length of one-eighth inch drill rod, threaded at the ends and supported on small rollers spaced at intervals of about eight inches to reduce friction between it and the gun and so ensure that the pull due to ex-

pansion of the gun is all transmitted to the contact. In mounting, the frame was placed on the gun where it was held from slipping by the four, pointed legs, two of which are shown in figure 2. The left end of the band was next screwed into the long turnbuckle as shown, which is held in the pivoted block. The right end was then inserted in the shorter turnbuckle and after the rollers were inserted and spaced this turnbuckle was adjusted until the rod carrying the grounded contact was pulled - against the force of the strong regulating spring - well clear of the forward end of the pivoted cylinder to a position about midway in its clearance. It should be noted that the grounded contact is mounted on a thin metal diaphragm instead of directly on the rod. The function of the diaphragm is to absorb any mechanical shocks or vibrations as the gun recoils and so ensure a steady contact.

Bolted to the frame so as to be substantially integral with it, is the fixed cylinder containing an oil chamber in which moves a piston backed up by a spring somewhat weaker than the regulating spring referred to above. Near the rear end of the piston rod is fastened a yoke which moves in clots on the top and bottom of the cylinder and carries the contactor mounted on a fibre block and so insulated from the remainder of the instrument. Motion of the piston is therefore transmitted directly to this contactor.

Through the piston, connecting the two ends of the oil chamber are two passageways, one at the center through which the flow of oil is regulated by the needle valve extending through the piston rod, and the other, shown above it with a ball valve

held closed by the oil pressure and a spring during forward motion of the piston but opening under the pressure of oil from the rear when the piston moves backward.

Because of the action of the spring, the piston always tends to move forward and maintain contact between the points. This motion is restrained by the presence of the oil in the chamber and can be stopped altogether by closing the needle valve completely. The speed of the forward motion is therefore regulated by means of the needle valve which in operation is adjusted so that the piston creeps forward almost imperceptibly when the contact is open. Motion to the rear however is but little affected by the presence of the oil because of the ease of oil flow forward through the by-pass above.

With this arrangement all the requirements set forth in the preceding pages are quite adequately met. If after initial adjustment, the gun for any reason contracts or expands slowly as it would by reason of temperature changes, contact between the points is maintained continuously, since the piston can follow any gradual forward motion of the contact and moves readily backward. When however the projectile passes and a sudden expansion of the section of the gun at the expansometer exerts a sudden pull on the rod bearing the contact, the piston is unable to follow this rapid motion and the contact is broken. It remains open until the gradual seepage of oil through the bleeding passage permits the contactor to overtake the contact or meet it returning as the reduction in pressure inside the gun eases the pull on the regulating spring. It should be noted here

that it is immaterial that contact will be restored while the points are displaced to the right of their initial position, because the fact that the regulating spring is the stronger of the two and that the piston is free to move backward, will result in a gradual restoration of the original conditions within the instrument, when the gun does contract to its original size.

In view of the very satisfactory operation of the expansometer during the present experiment a word or two in a historical sense, concerning its development may not be amiss. The possibilities and advantages of measuring projectile displacement by this method were first suggested by Dr. H. L. Curtis of this Bureau in 1919 and a first design was completed and tried out in the following year. Because of mechanical difficulties the early tests were unsuccessful. The development of the instrument has been continued ever since as occasion offered, but it was not until early in 1923 that satisfactory records were obtained. During the latter half of 1922 and the first half of 1923, the development work was carried on by Mr. H. H. Moore now of the Naval Research Laboratory and it is to his efforts that the chief features of the present design are to be credited.

7. Number and Location of Expansometers.

During the present experiment eight expansometers were mounted on the gun thus giving a record of the passage of the shell at eight positions along the bore. In order to make best use of the capacity of the recording apparatus, two expansometers were connected in parallel to each of four oscillograph elements. The opening of the first of a pair so connected doubled the

resistance of the circuit to ground and halved the oscillograph deflection, while the breaking of the contact in the second, opened the circuit completely and caused the deflection to drop to zero. In order to be sure that the self-closing feature of the earlier acting expansometer could not by premature closing interfere with the record of the one used with it, a relay was connected in series with the first of each pair in such a way as to take the instrument out of circuit as soon as it opened thus leaving the current in the oscillograph circuit solely under control of the second expansometer of the pair. The connection showing the pairing of the expansometers are given in the recording circuit wiring diagram figure 3 and their position along the bore of the gun is listed in the following table.

Expansometer Number	Distance back from muzzle	Travel of Projectile before Expansometer acts.	Oscillograph Film
	Feet	Feet	
1	33.63	16.56	B 3 (Relay)
2	27.54	22.65	C 3 (Relay)
3	21.86	28.33	D 2 (Relay)
4	18.74	31.45	D 3 (Relay)
5	14.38	35.83	B 3
6	11.45	38.74	C 3
7	6.86	43.33	D 2
8	3.18	47.01	D 3

8. Recoil Displacement and Recoil Forces.

With reference to the measurements and apparatus listed under "Recoil", the recoilmeter and the recoil cylinder pressure gages are familiar pieces of apparatus and require no detailed description. The instruments were the same as those used in previous experiments and described in former reports. The determination

in the field, however, of the other forces acting during recoil, viz. the force of friction between the gun and slide and the force exerted by the counter recoil springs, has not hitherto been attempted by this Bureau and the method by which data on these factors were obtained is described below.

9. Measurement of Spring Force and Friction Between Gun and Slide.

In order to determine the force of the springs and of friction advantage was taken of the fact that, unless assisted by the introduction of compressed air into the counter recoil cylinders, the springs are not sufficiently strong to hold the gun in battery at any great elevation. Because of this fact no elaborate apparatus was required for the measurement. The gun was slowly elevated by hand until it just started to slide out of battery and the elevation at which this took place was determined by the use of a quadrant. Since the only forces acting on the gun under these conditions are gravity, the springs and the friction between gun and slide, the equilibrium condition when the gun is just on the point of sliding out of battery under its own weight may be represented by the equation

$$S_o + F_o \cos \theta_o = W \sin \theta_o$$

where S_o = Initial force of springs in pounds.

F_o = Force of friction in pounds between gun and slide at 0° elevation.

W = Weight of the recoiling parts of the gun in pounds.

θ_o = Angle at which the gun is just on the point of leaving battery, measured by quadrant.

Suppose that the gun is now elevated a little more, still very slowly to minimize the effect of momentum. Under these

conditions the gun will slide out of battery until the increased resistance offered by the springs as they compress balances the increase in the value of the right-hand side of the equation due to the greater angle of elevation. The equilibrium equation then becomes

$$S_o + Kl_1 + F_o \cos \theta_1 = W \sin \theta_1$$

where K = spring constant in pounds per unit distance

l_1 = distance in inches which the gun has moved out of battery

θ_1 = the angle of elevation

and so S_o, F_o and W_o are the same as before.

Hence by elevating the gun slowly to a number of different angular positions and measuring θ and l each time the gun came to rest a sufficient number of simultaneous equations were secured to determine and adequately check the values of the three unknown quantities S_o , K and F_o.

Two runs were made before the experimental firing, the gun being slowly elevated by hand in order to prevent the equilibrium conditions from becoming indeterminate by reason of the effect of the momentum acquired by the gun on the position in which it came to rest. Elevation was made in steps of about one degree and continued until the gun had slid about 10 inches out of battery. Readings were taken at each step and also on the return run as the gun was depressed slowly by intervals until it returned to battery. Since the friction always acts in a direction opposing $S_o + Kl_n - F_o \cos \theta_n = W \sin \theta_n$ for positions out of battery and $S_o - F_o \cos \theta_r = W \sin \theta_r$ for the angle at which the gun

just slides back into battery.

Following the completion of the firing a check run was made in the same manner.

Although the procedure outlined above sufficed for the determination of both spring force and friction, an independent measurement of the friction was made by slacking off the bolts fastening the counter recoil piston to the gun and measuring the angle at which the gun started to slide out of battery when its motion was opposed by the force of friction alone. This afforded a splendid check on the previous determinations and a more accurate measurement of the friction.

10. The Recording Circuit.

A wiring diagram showing the distribution of the apparatus among the four oscillographs used in the experiment is given in figure 3. The ejection velocity and all internal velocity records were grouped as shown in a single oscillograph (A) the three elements of which were connected to a recoil operated common time device to provide an accurate means of correlating the individual determination of ejection velocity. For a common time record on the other three oscillographs three additional muzzle fingers of the ogive type were used so that the ejection of the ogive of the projectile was recorded by all four oscillographs.

11. Operating Circuit.

The operating circuit used for the experiment is shown in figure 4. Oscillographs "B" "C" and "D" all of which recorded exclusively events occurring before ejection, were connected in

parallel in the usual manner so that their shutters were opened by the closing of the firing key just before the firing circuit was closed through a delay relay. The shutter of oscillograph "A" and that of the camera were opened by the action of a switch which was closed by the start of recoil of the gun. By thus delaying the opening of the shutter of oscillograph "A" which, as is shown in figure 3, was not required to record any event before ejection, it was possible to keep the shutter of the oscillograph open until after the projectile had passed through the second contact screen and completed the external velocity records, without reducing the speed at which the film drum was operated.

12. Location of the Camera.

The camera was located exactly under the second contact screen at a horizontal distance of 160.9 feet from the muzzle of the gun. It pointed vertically upward and included in the field the contact screen which was equipped with two arms extending across the trajectory of the projectile, perpendicular to it and the line of motion of the camera film. Each arm bore a reference mark which was photographed by the camera and afforded a means of determining the ratio of magnification "r" used in calculating the velocity of the projectile from its photographs on the camera film.

13. Fork Calibration.

In order to be sure of the greatest possible accuracy for the time measurement, the tuning forks used in the four oscillographs were calibrated immediately after the test. The frequency

of the fork in oscillograph "A" was measured by the drop chronograph method of Dr. L. T. E. Thompson and found to have a frequency of 500.7 cycles per second. The frequencies of the remaining three forks were then determined by comparing them with the fork in oscillograph A, and they were found to be 500.2, 499.8, and 499.8, respectively for oscillograph, B, C and D. In interpreting the results of the velocity determinations allowance was made for the difference between the actual frequency 500.7 cycles and the nominal frequency 500 cycles of the fork on oscillograph A. In the case of the other oscillographs the precision of the measurement was not sufficiently great to require that the small difference between the actual and nominal frequency be taken in account.

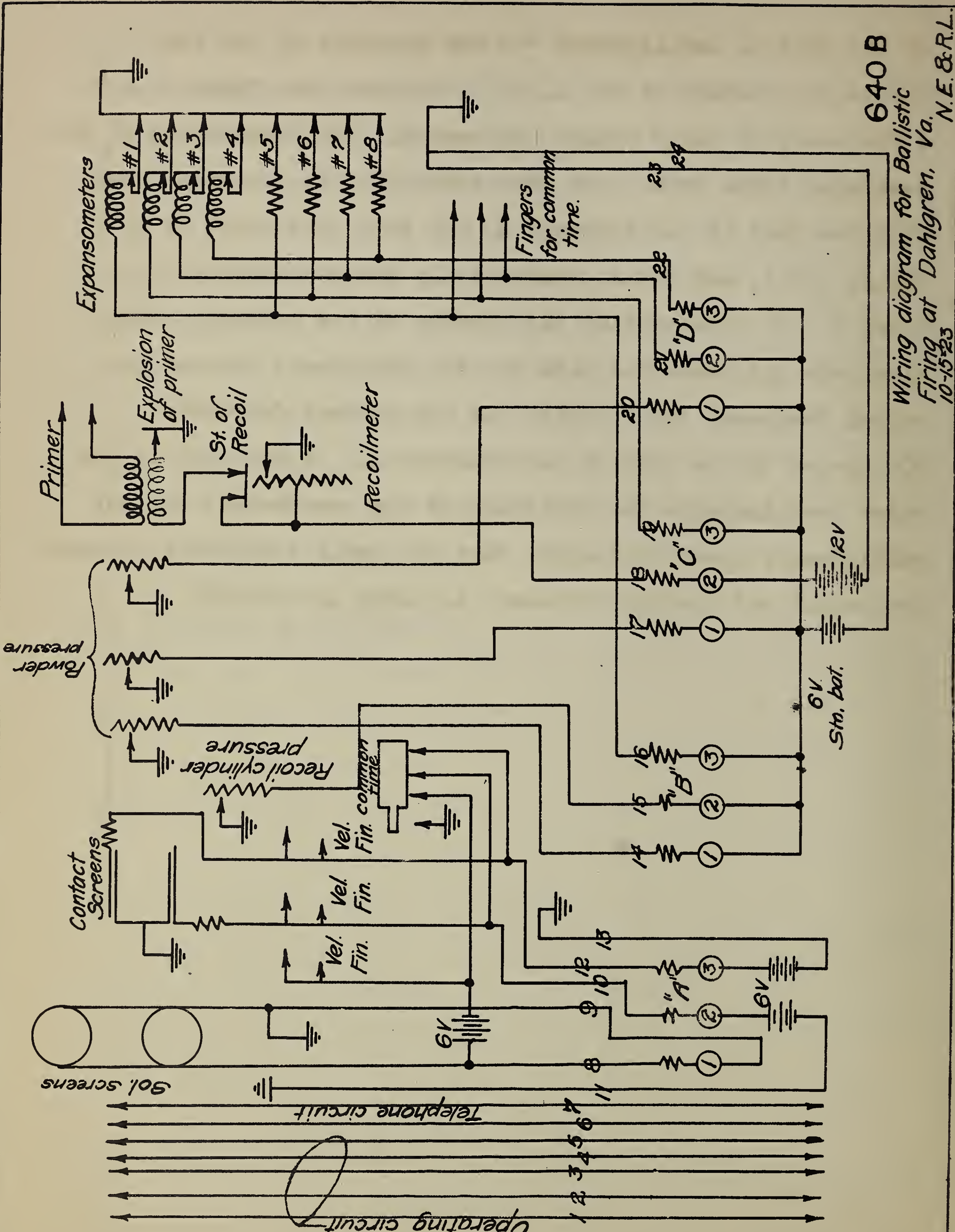
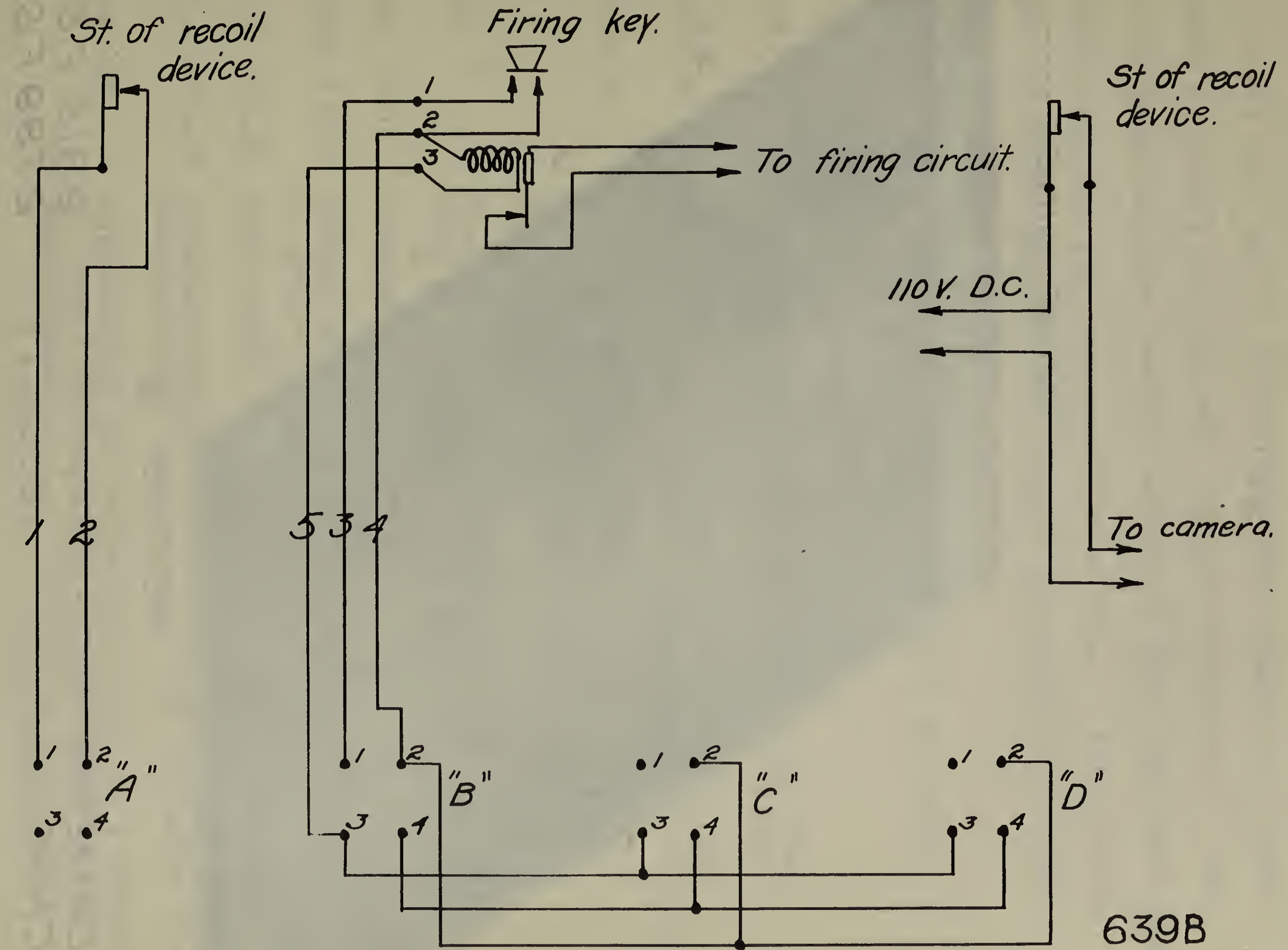

640 B
Wiring diagram for Ballistic
Firing at Dahlgren, Va.
10-15-23
N.E.&R.L.
Expansometers
#1
#2
#3
#4
#5
#6
#7
#8
Fingers
for common
time.
Primer
Explosion
of primer
St. of
Recoil
Recoilmeter
Powder
pressure
Recoil cylinder
pressure
Fin. common
time
Contact
Screens
Vel.
Fin.
Vel.
Fin.
Vel.
Fin.
Sol. screens
6V
6V
12V
6V.
Sto. bat.
"A"
"B"
"C"
"D"
Telephone circuit
Operating circuit

FIG. 3



639B

Operating circuit for Ballistic Firing at Dahlgren, Va. N.E.&R.L.
10-15-23.

FIG. 4

ROUND 2

CAMERA VELOCITY 2815 F.S.
CHRONOGRAPH VELOCITY 2799 F.S.

III. DATA AND RESULTS

In the five tables and thirty-three sheets of curves, on pages 25 to 62, all the more important features of the data collected and the results calculated from them have been gathered together for convenient reference. A list of the tables and curve sheets with a brief description of their contents follows:

Table 1.	Firing record. Condensed from the usual proving ground form.
Table 2.	Projectile data.
Table 3.	Star gage record of condition of gun 89-L2 made just before the experimental firing.
Table 4.	Velocity of projectile. Results obtained by the several methods of measurement employed.
Table 5.	Maximum powder pressure. Comparison of crusher gage records with those of time-pressure gages and with values calculated from recoil data.
Fig. 5	Graphical representation of condition of bore of gun constructed from star gage records.
Figs. 6-12	Time-pressure curves.
Figs. 13-19	Motion of gun in recoil up to time of ejection of projectile.
Figs. 20-25	Time curves of pressure in recoil cylinder.
Figs. 26-27	Pressure-displacement curves for recoil cylinder. Comparison of time records with indicator diagrams.
Fig. 28	Displacement of projectile in gun as a function of recoil from records of expansometers and recoilmeter.

Figs. 29-33	Motion of projectile in gun. Comparison of results obtained by direct measurement by expansometers and indirect determination by calculation from records of gun recoil and recoil forces. For rounds 1, 2, 3, 5, and 6.
Figs. 34-37	Force-time curves showing the force exerted by the powder gases as determined by time-pressure records and calculated from recoil data.

TABLE I

FIRING RECORD

Six Experimental Rounds for Bureau of Standards Test
Naval Proving Ground, Dahlgren, Va., October 1923
14"-50 Cal. Gun Mark IV Mod. 3, No. 89-L2
Uniform Rifling 1/32

Experimental Round Number	When Fired Date Oct.	Time	Weight of Charge lbs.	Projectiles No. on Shell	Seating Inch.	Chronograph Velocity Ft/Sec.	Crusher Gage Pressure Tons/ Sq.In.	Range Yards	Drift Yards
1	26	15:42	484	856	97.6	2799	15.65	16130	128 R
2	27	11:36	484	1274	97.7	2799	15.75	16057	136 R
3	29	14:37	484	964	97.7	2799	15.21	15767	71 R
4	30	10:08	230	1213	97.7	1478	3.42	5452	3 L
5	30	12:45	368	1338	97.6	2172	7.89	10578	28 R
6	30	14:11	518	1210	97.6	2997	19.14	17546	68 R
7*	30	15:43	240	1218	97.6	1541	3.76	5912	4 R

Powder: Rounds 1 - 5 & 7 S.P.D. 2082
Round 6 S.P.D. 1563

Loading: 4 Silk Bags, stacked for Rounds 1, 2, 3, and 6; dumped for Rounds 4, 5, and 7.

Ignition: 1200 gms. Black Powder, distributed 300 gms per section.

Projectiles: F.S.S.Co. A.P. Mk 5, Mod. 2B, Lot 4. Modified Band Sketch No. 37088; for individual shell measurements see Table 2.

For all Rounds: Elevation, 8°; flight smooth.

*Round 7 was an additional round fired to replace Round 4 in which screen velocities were lost.

DIMENSIONS OF PROJECTILES

Used in Experimental Firing, Naval Proving Ground,
Dahlgren, Virginia, October 26-30, 1923.

A.P.Shell Mk V Mod. 2-B, Lot 4, Contract 10, Modified Band.
Length of Cap 17.2 Diameter of Base of Cap 11.572

(Dimensions in inches unless otherwise specified)

Number		Used in Round No.	Weight Lbs.	Length Inches	Diameter			Effect. Length For Ejection Velocity Measurements. By Special Gage
On Shell	On Band				Bourrelet	Body	Band	
856	R5	1	1400	49.4	13.980	13.939	14.170	24.61
1274	R4	2	1400	49.4	13.979	13.950	14.169	24.50
984	R3	3	1400	49.4	13.981	13.943	14.169	24.47
1213	R2	4	1400	49.4	13.983	13.948	14.169	24.54
1338	R1	5	1400	49.4	13.978	13.938	14.173	24.51
1210	R6	6	1400	49.4	13.982	13.948	14.170	24.50
1218	U1	7	1400	49.4	13.973	13.940	14.173	24.60

For all rounds: Width of Band - 4.7

Distance Band to Base - 1.1

TABLE 3

STAR GAGE RECORD

14"-50 Cal. Gun Mark IV, Model 3, No. 89-L2
October 10, 1923

Previous Actual Rounds - 195
Previous Equivalent Service Rounds - 124.55

1		2		3	
Distance Forward of Origin of Rifling Inches	Diameter Inches	Distance Forward of Origin Inches	Diameter Inches	Distance Forward of Origin Inches	Diameter Inches
0 at Origin	14.275	203.33	13.998	443.33	14.073
1	14.240	213.33	.994	453.33	.081
12	.206	223.33	.992	463.33	.086
13.33	.203	233.33	.990	473.33	.088
18.33	.192	243.33	.986	483.33	.092
23.33	.181	253.33	.993	488.33	.092
28.33	.168	263.33	.989	493.33	.092
33.33	.155	273.33	.990	498.33	.092
43.33	.133	283.33	.991	503.33	.094
53.33	.113	293.33	.994	513.33	.097
63.33	.094	303.33	.995	523.33	.100
73.33	.077	313.33	.993	533.33	.105
83.33	.063	323.33	13.995	543.33	.109
93.33	.049	333.33	14.000	553.33	.114
103.33	.036	343.33	14.005	563.33	.120
113.33	.027	353.33	.011	573.33	.128
123.33	.017	363.33	.016	578.33	.133
133.33	.010	373.33	.024		
143.33	.006	383.33	.030	583.33*	14.134
153.33	.002	393.33	.037		
163.33	.001	403.33	.048		
173.33	14.000	413.33	.055		
183.33	13.999	423.33	.062		
193.33	13.996	433.33	.067		

* Muzzle

TABLE 4

VELOCITY OF PROJECTILE

Results Obtained by Different Methods of Measurement
Experimental Firing of 14"-50 Cal. Gun No. 89-L2
Naval Proving Ground, Dahlgren, Va., October - 1923

	Method of Determination	Velocity Feed per Second Round 1	Round 2	Round 3	Round 4	Round 5	Round 6	Round 7
1	Contact screens	2802	2798.5	2790	lost	2167	lost	1536
2	Solenoids	2801	2789	2786	10st	2168.5	lost	1532.5
3	Rot. band finger & first contact screen	2799	2794	2785	1493	2168.5	2978	1537
4	Boulenge chronograph (average of three)	2799	2799	2799	1482	2172	2997	1541
5	Average of 1, 2, 3, and 4	2800	2795	2790	-	2169	-	1537
6	Muzzle Fingers (Pair 1	2796+	2768	2841+	1455	2136	lost	1521
7	(Pair 2	2782	2784	2753	1485	2159	lost	1532
8	(Pair 3	2773	2740	2763	1443	2129	lost	1509
9	(Average	2783	2764	2785	1462	2141	-	1521
10	Camera	-	2815	2800	-	-	-	-
11	From curves of shell motion inside gun. Figures 29-33	2814	2820	2822	-	2127	3037	-
12	Difference between contact screen and average in line 5	+2	+3.5	0	-	-2	-	-1
13	Difference between solenoids and average in line 5	+1	-7	-4	-	-0.5	-	-4.5
14	Difference between Rot. band finger-contact screen method and average in line 5	-1	-1	-5	-	-0.5	-	0
15	Difference between Boulenge chronograph and average in line 5	-1	+4	+9	-	+3	-	+4
16	Difference between finger average and average in line 5	-17	-31	-5	-	-24	-	-16
17	Maximum difference between records by individual pairs of fingers	23	44	78	42	30	-	22

TABLE 3

MAXIMUM PRESSURE
Experimental Firing of 14"-50 Cal. Gun No. 89-L2
Naval Proving Ground, Dahlgren, Virginia,
October 26-30, 1923

Round Number	Weight of Charge Pounds	Chronograph Velocity Ft./Sec.	Maximum Pressure Lbs. per Sq. In. Crusher Gages	Time Pressure Gage	Calculated From Recoil Data*
1	484	2799	35100	37300	44500
2	484	2799	35300	37200	41300
3	484	2799	34100	36100	43900
4	230	1478	7700	8400	Not calculated
5	368	2172	17700	20700	" "
6	518	2997	42900	45500	80600
7	240	1541	9400	9800	Not calculated

*Values in this column were calculated from the records of recoil by the approximate formula

$$P = \frac{M\alpha + F_1 + F_2 + F_3}{a}$$

where

α = acceleration of recoiling parts

M = mass of recoiling parts

F_1 = force of friction between gun and slide

F_2 = force exerted by counter-recoil springs

F_3 = force exerted by recoil cylinder pressure

a = area of bore of gun

The force of friction between shell and gun which could not be measured was neglected in the calculations.

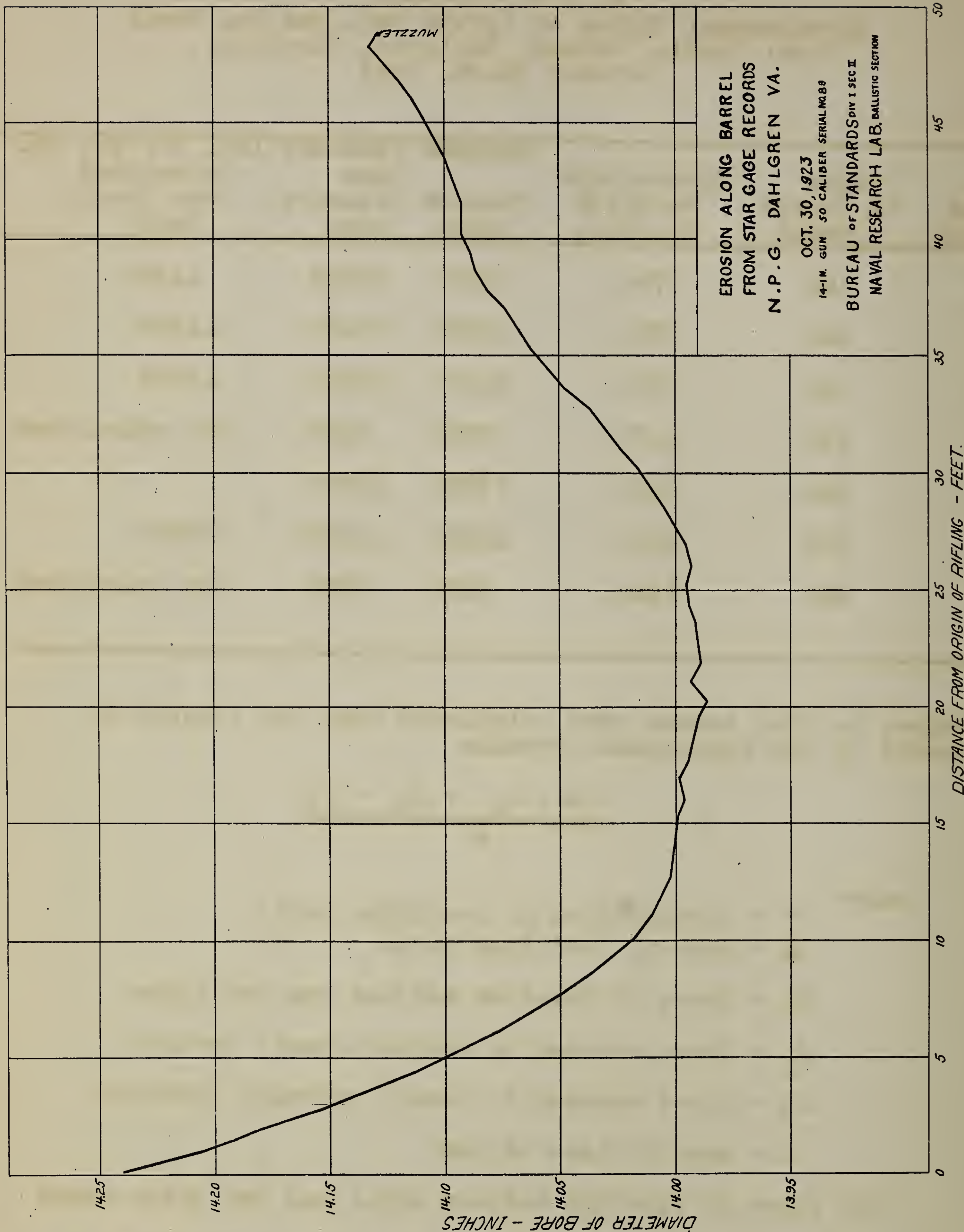

EROSION ALONG BARREL
FROM STAR GAGE RECORDS
N.P.G. DAHLGREN VA.
OCT. 30, 1923
14-IN. GUN 50 CALIBER SERIAL NO. 89
BUREAU OF STANDARDS DIV I SEC II
NAVAL RESEARCH LAB. BALLISTIC SECTION
MUZZLE
DISTANCE FROM ORIGIN OF RIFLING - FEET.
DIAMETER OF BORE - INCHES
0
5
10
15
20
25
30
35
40
45
50
14.25
14.20
14.15
14.10
14.05
14.00
13.95

FIG. 5

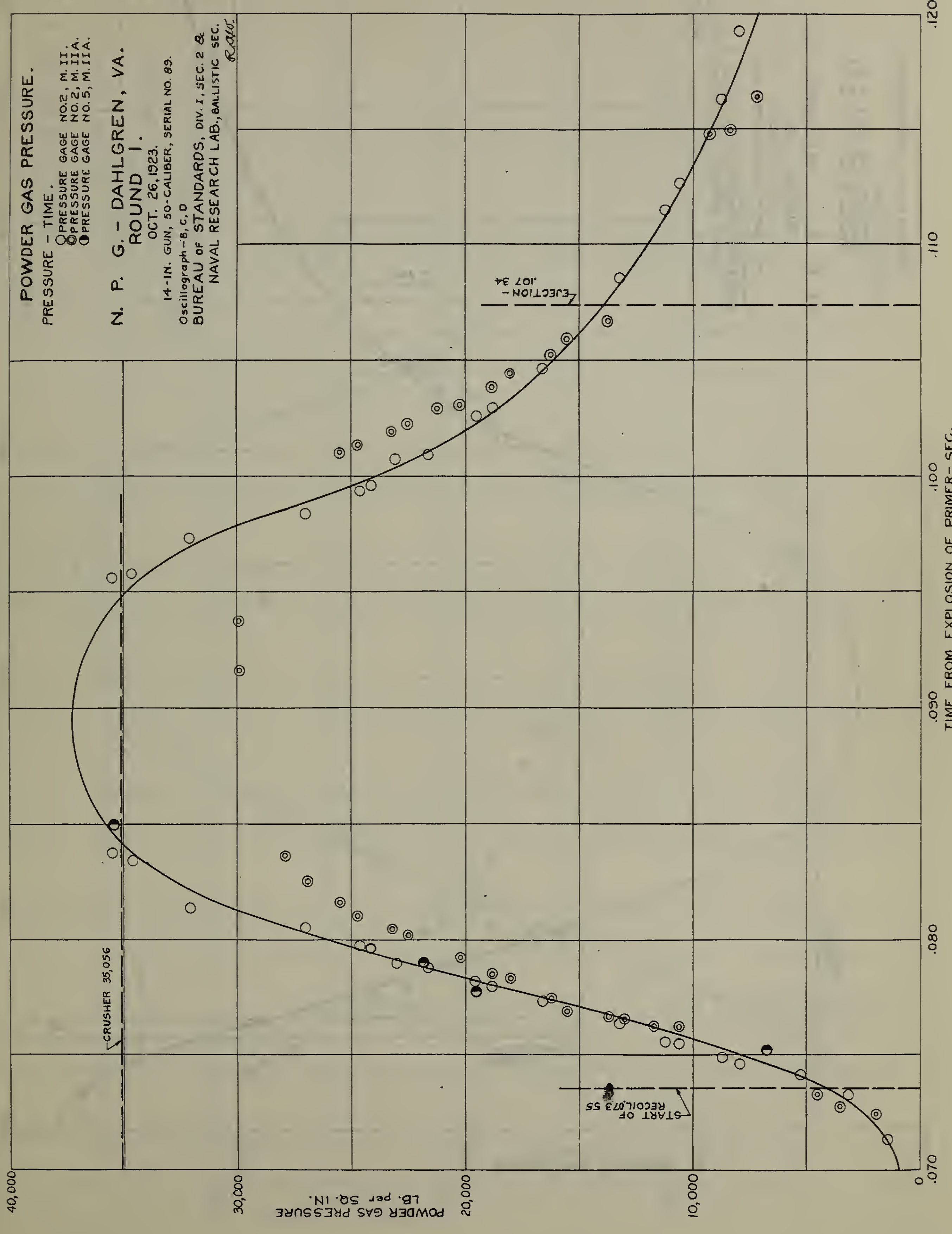
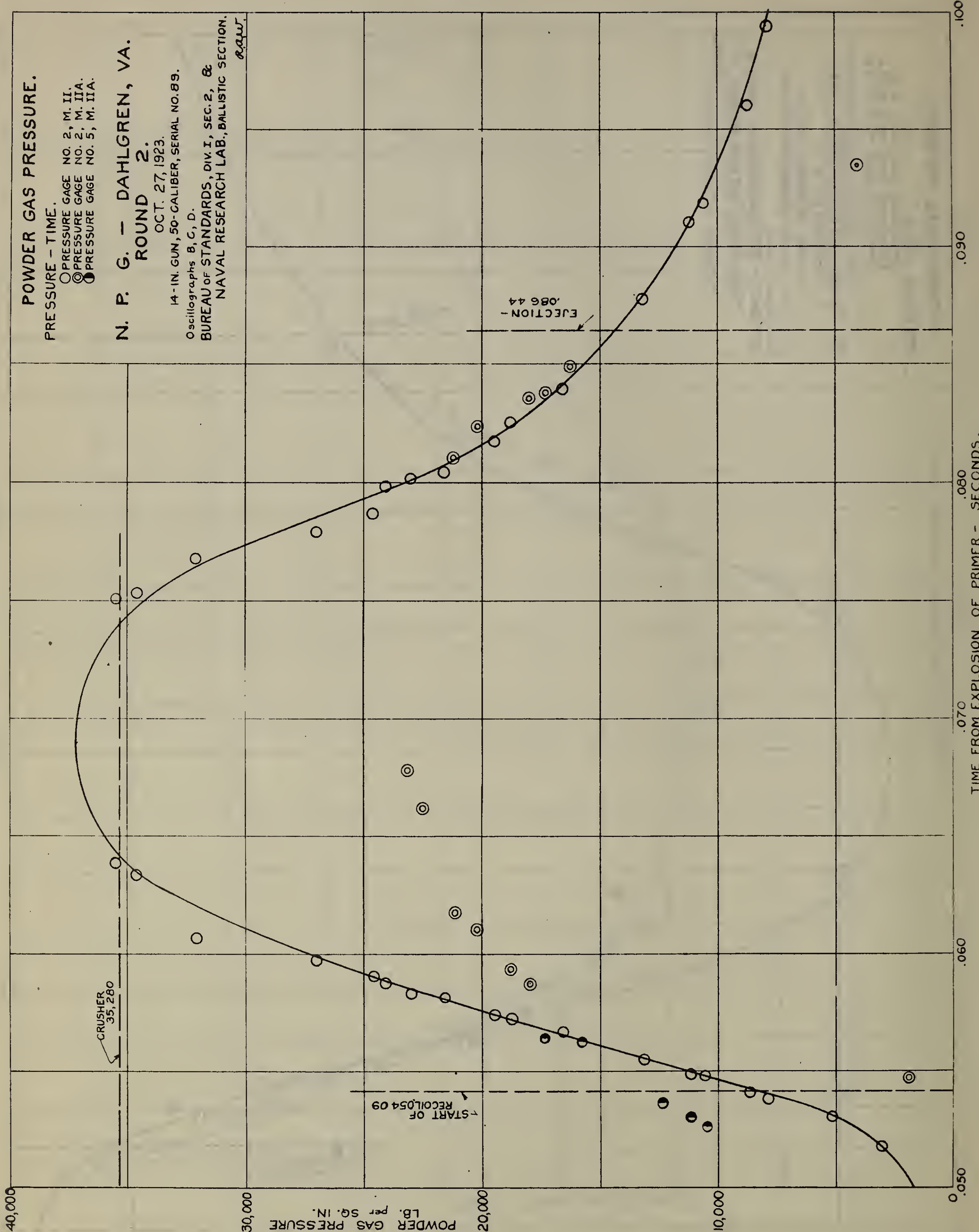
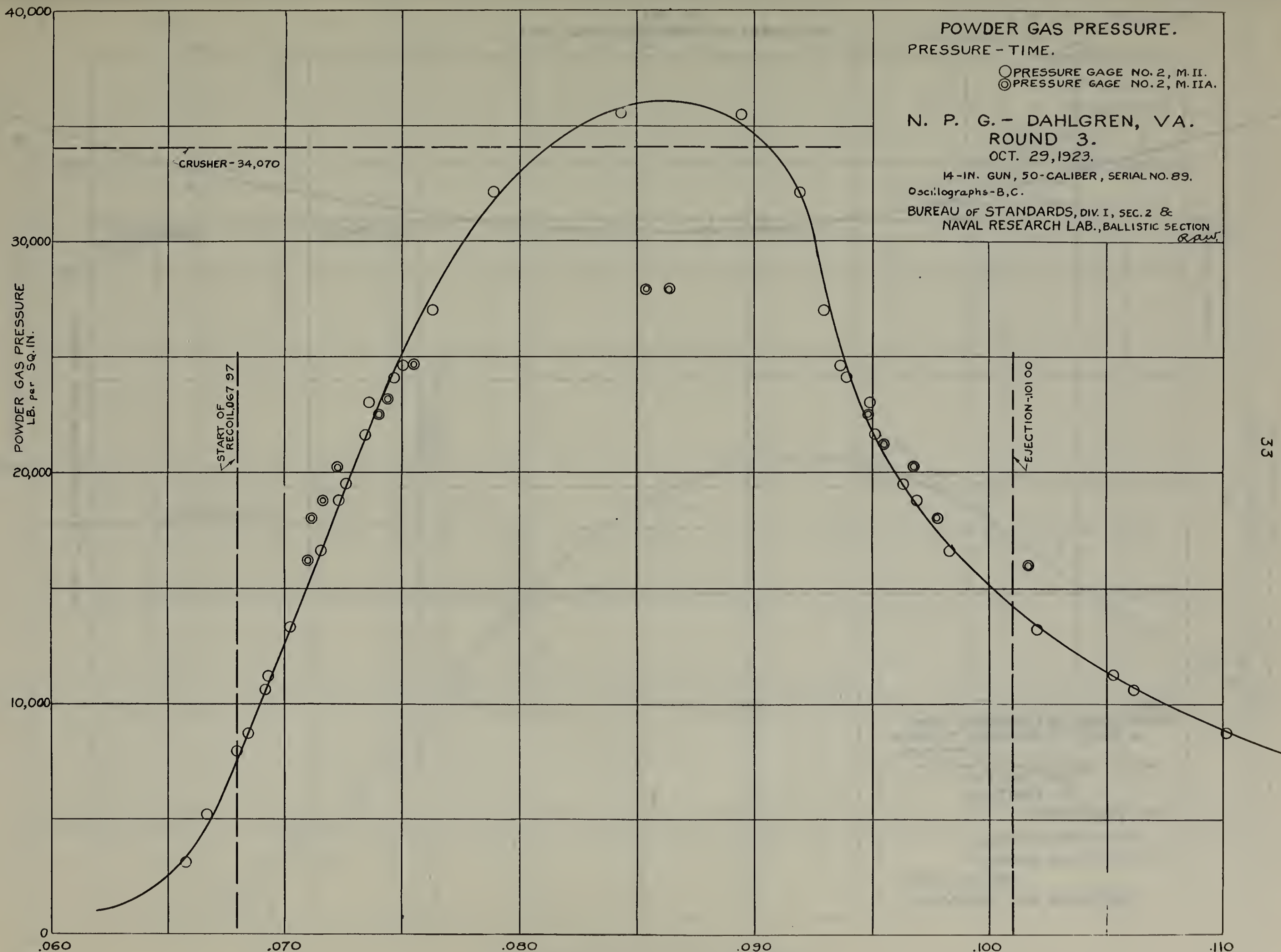

POWDER GAS PRESSURE.
PRESSURE - TIME.
PRESSURE GAGE NO.2, M. II.
PRESSURE GAGE NO.2, M. IIA.
PRESSURE GAGE NO.5, M. IIA.
N. P. G. - DAHLGREN, VA.
ROUND 1.
OCT. 26, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
Oscillograph - B, C, D
BUREAU of STANDARDS, DIV. I, SEC. 2 &
NAVAL RESEARCH LAB., BALLISTIC SEC.
CRUSHER 35,056
START OF RECOIL .073 55
EJECTION - .107 34
POWDER GAS PRESSURE LB. per SQ. IN.
40,000
30,000
20,000
10,000
0
.070
.080
.090
.100
.110
.120
TIME FROM EXPLOSION OF PRIMER - SEC.

FIG. 6



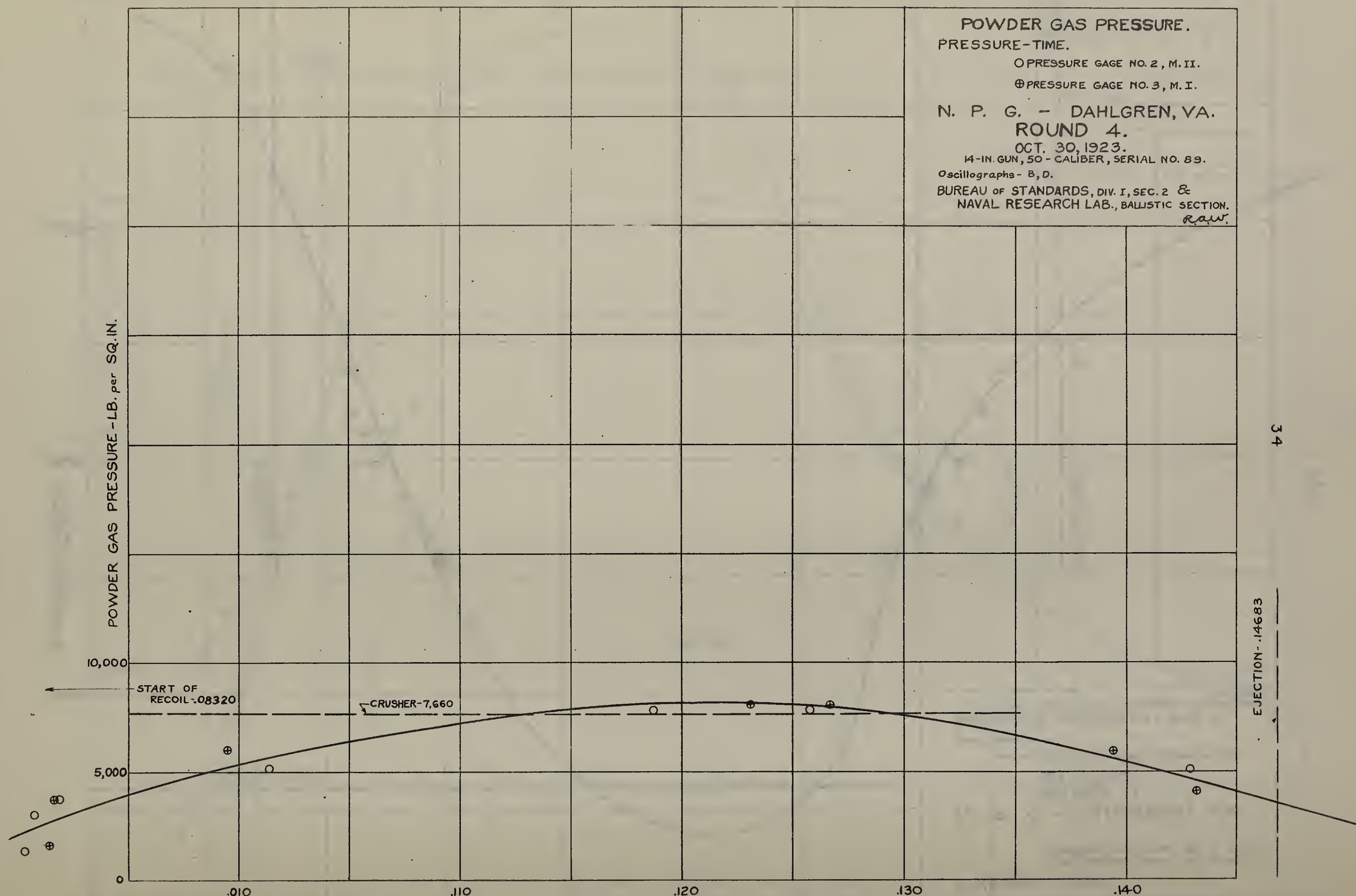
POWDER GAS PRESSURE.
PRESSURE - TIME.
PRESSURE GAGE NO. 2, M. II.
PRESSURE GAGE NO. 2, M. IIA.
PRESSURE GAGE NO. 5, M. IIA.
N. P. G. — DAHLGREN, VA.
ROUND 2.
OCT. 27, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
Oscillographs B, C, D.
BUREAU OF STANDARDS, DIV. I, SEC. 2, &
NAVAL RESEARCH LAB., BALLISTIC SECTION.
CRUSHER 35,280
START OF RECOIL .054 09
EJECTION - .086 44
POWDER GAS PRESSURE LB. per SQ. IN.
40,000
30,000
20,000
10,000
0
.050
.060
.070
.080
.090
.100
TIME FROM EXPLOSION OF PRIMER - SECONDS.

FIG. 7



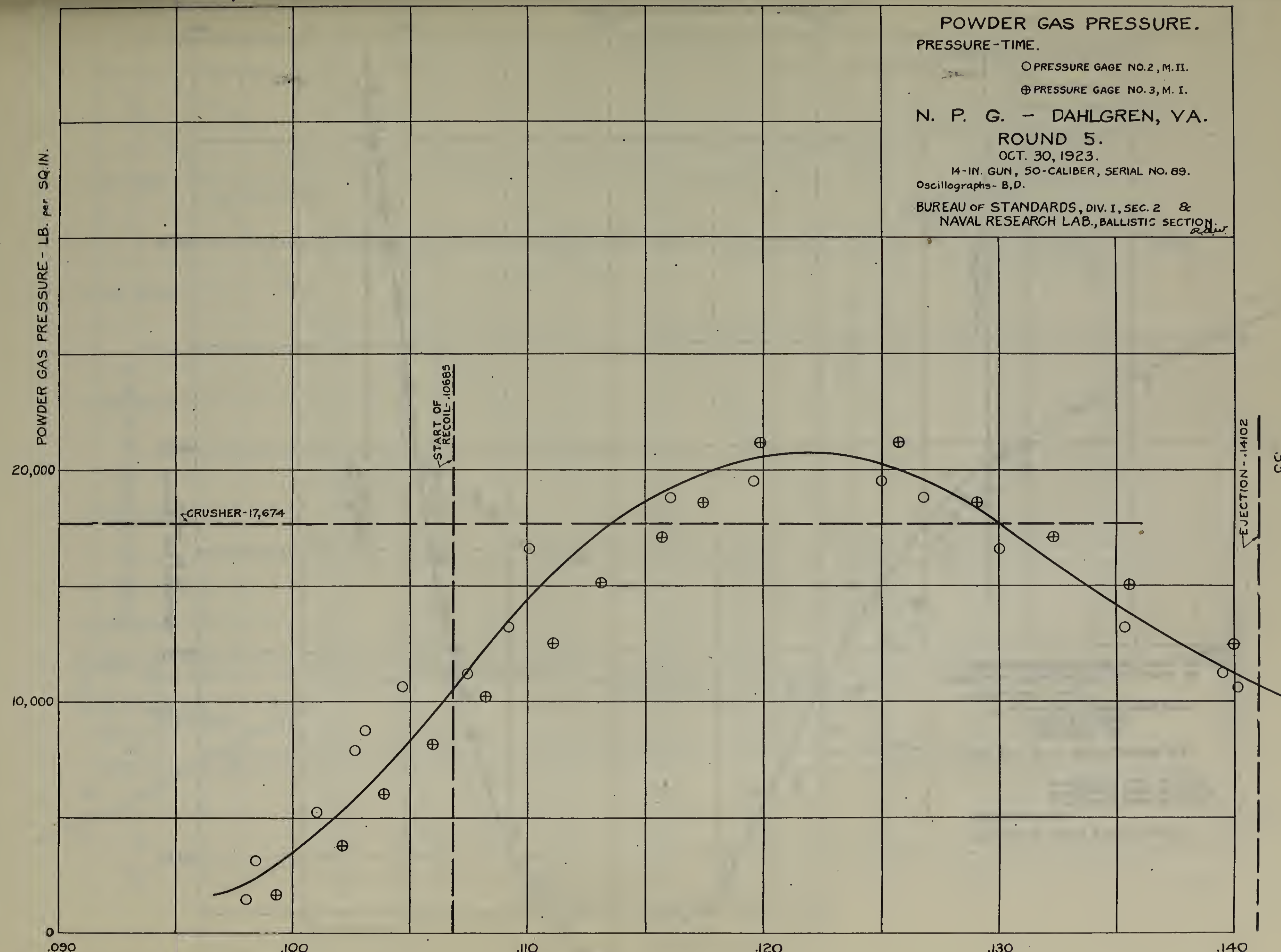
POWDER GAS PRESSURE.
PRESSURE - TIME.
○PRESSURE GAGE NO. 2, M. II.
◎PRESSURE GAGE NO. 2, M. IIA.
N. P. G. - DAHLGREN, VA.
ROUND 3.
OCT. 29, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
Oscillographs-B, C.
BUREAU OF STANDARDS, DIV. I, SEC. 2 &
NAVAL RESEARCH LAB., BALLISTIC SECTION
CRUSHER - 34,070
START OF RECOIL .067 97
EJECTION - .101 00
POWDER GAS PRESSURE
LB. per SQ. IN.
40,000
30,000
20,000
10,000
0
.060
.070
.080
.090
.100
.110
TIME FROM EXPLOSION OF PRIMER - SEC.

FIG. 8



TIME FROM EXPLOSION OF PRIMER - SEC.

FIG. 9



TIME FROM EXPLOSION OF PRIMER - SEC.

FIG. 10

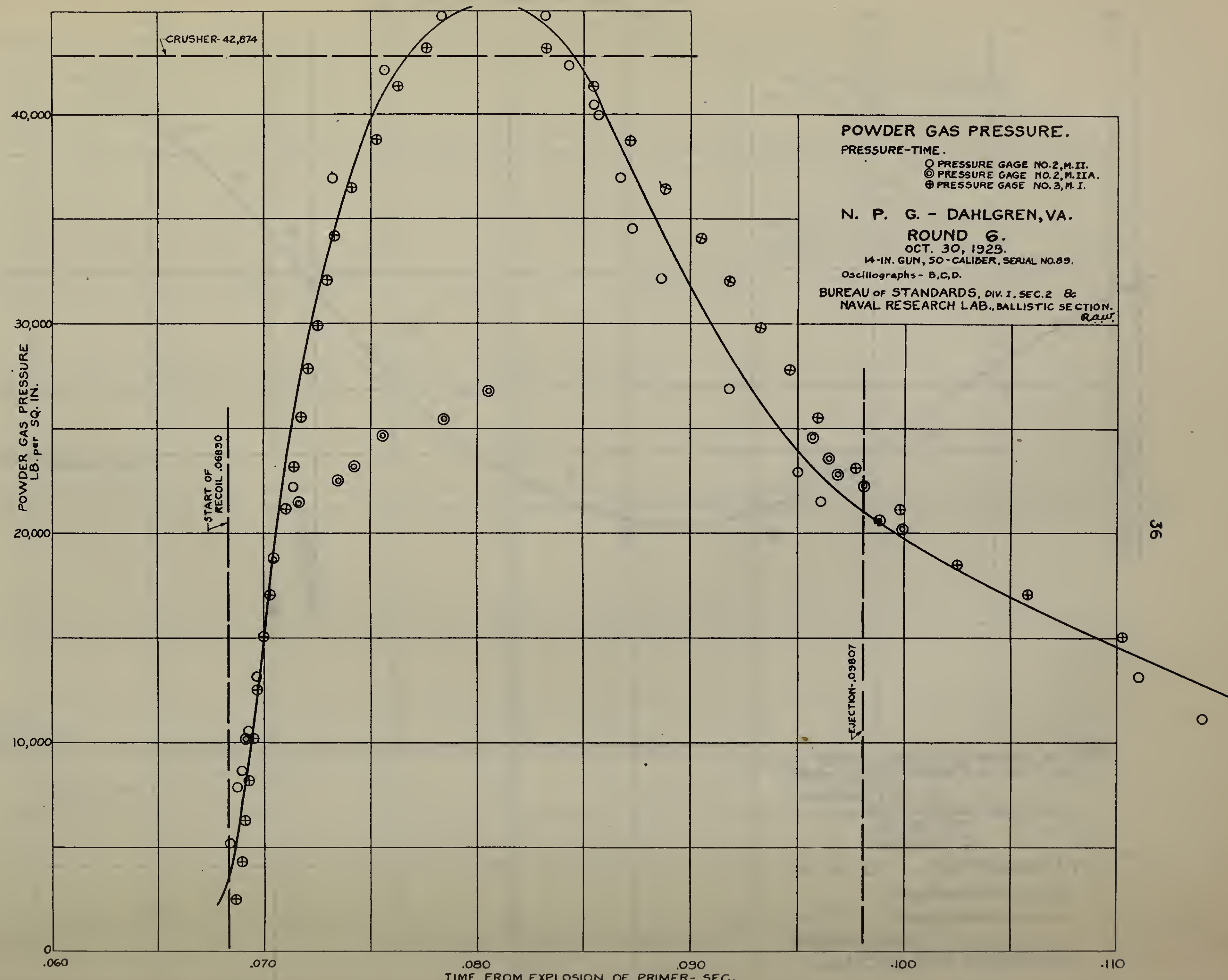

CRUSHER-42,874
POWDER GAS PRESSURE.
PRESSURE-TIME.
PRESSURE GAGE NO.2, M.II.
PRESSURE GAGE NO.2, M.IIA.
PRESSURE GAGE NO.3, M.I.
N. P. G. - DAHLGREN, VA.
ROUND 6.
OCT. 30, 1929.
14-IN. GUN, 50-CALIBER, SERIAL NO.89.
Oscillographs - B,C,D.
BUREAU of STANDARDS, DIV. I, SEC.2 &
NAVAL RESEARCH LAB., BALLISTIC SECTION.
Raw.
START OF RECOIL .06830
EJECTION-.09807
POWDER GAS PRESSURE
LB. per SQ. IN.
40,000
30,000
20,000
10,000
0
.060
.070
.080
.090
.100
.110
TIME FROM EXPLOSION OF PRIMER- SEC.

FIG. 11

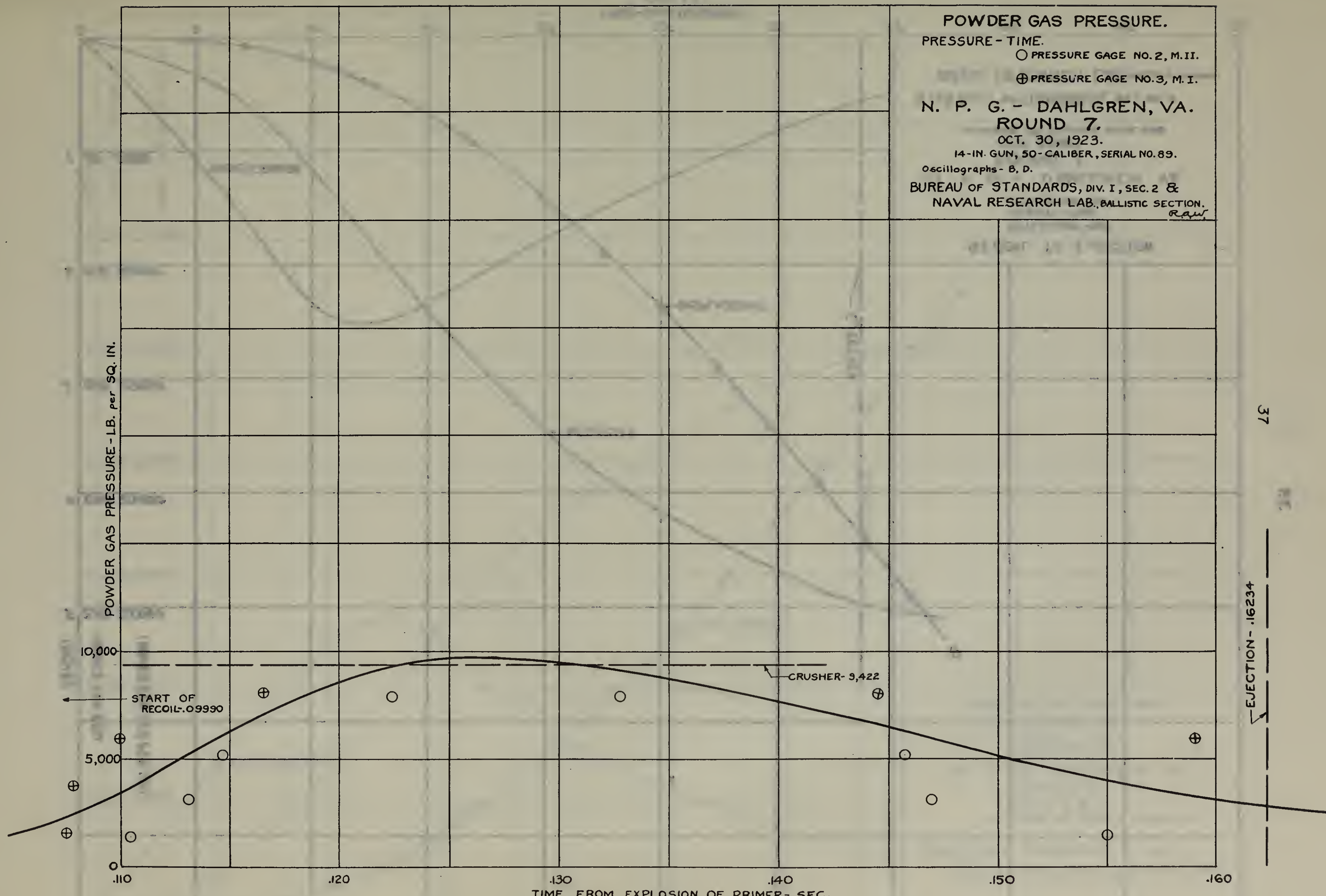
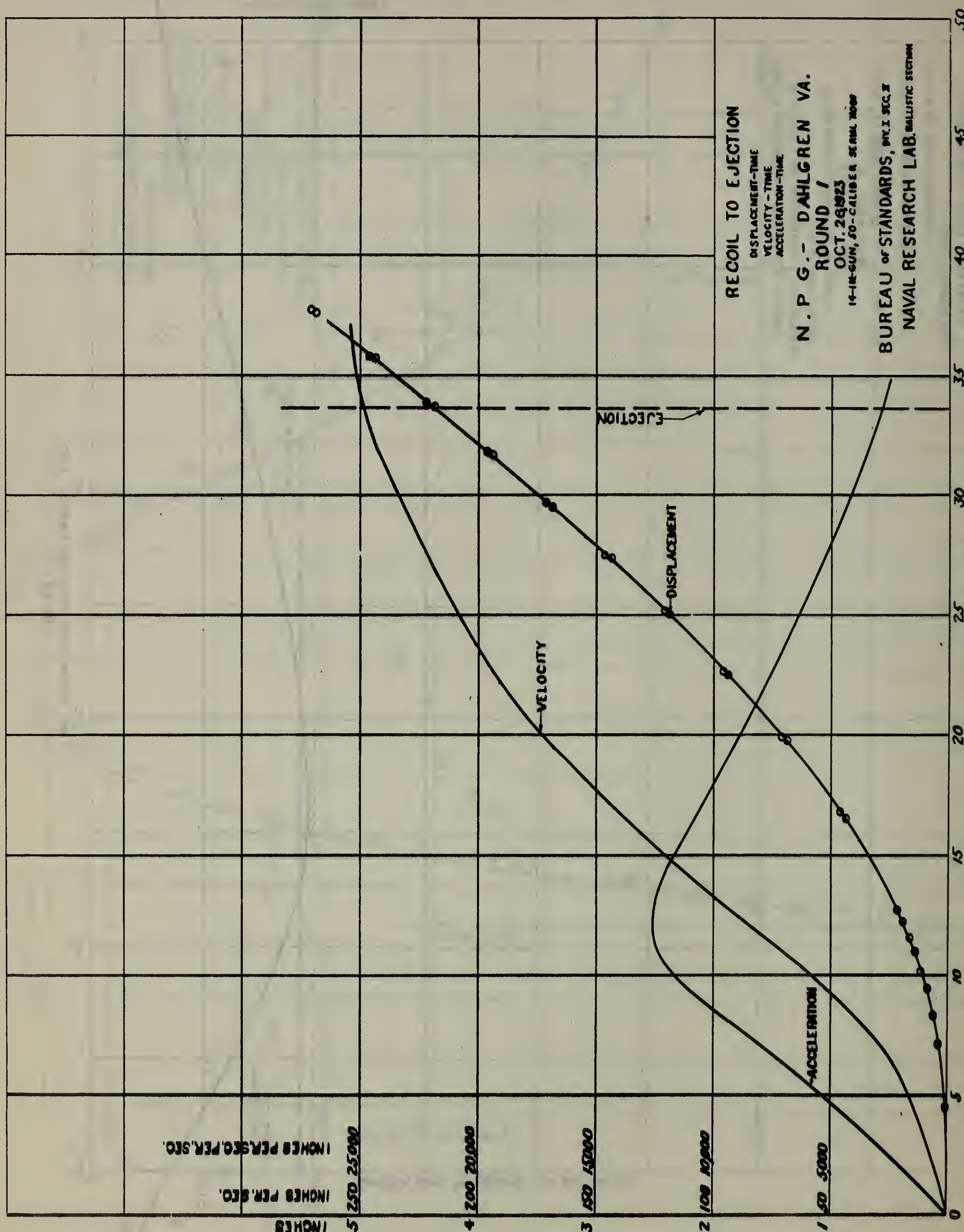
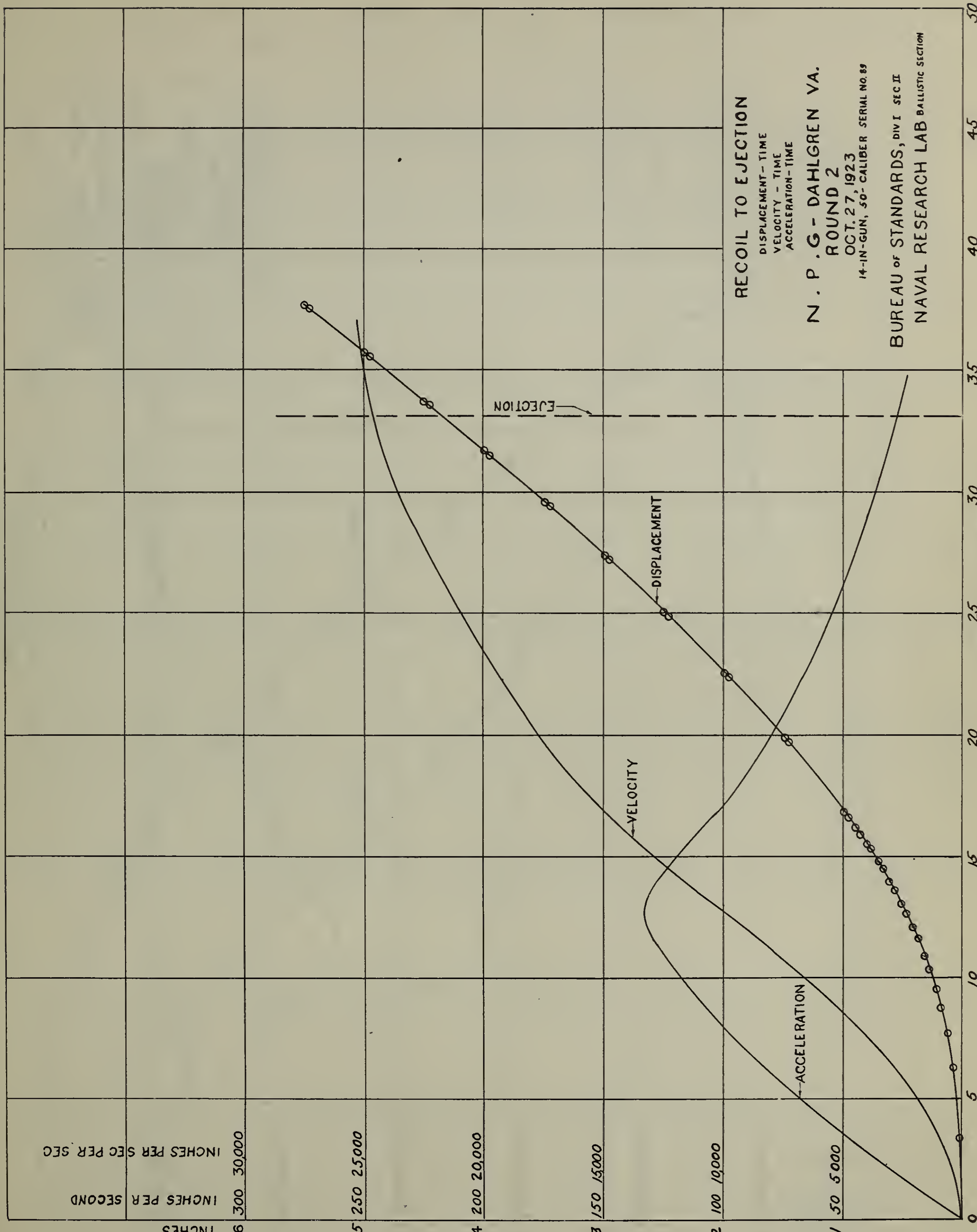

POWDER GAS PRESSURE.
PRESSURE - TIME.
○ PRESSURE GAGE NO. 2, M. II.
⊕ PRESSURE GAGE NO. 3, M. I.
N. P. G. - DAHLGREN, VA.
ROUND 7.
OCT. 30, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
Oscillographs - B, D.
BUREAU OF STANDARDS, DIV. I, SEC. 2 &
NAVAL RESEARCH LAB., BALLISTIC SECTION.
RAW
POWDER GAS PRESSURE - LB. per SQ. IN.
10,000
5,000
0
START OF RECOIL - .09990
CRUSHER - 9,422
EJECTION - .16234
.110
.120
.130
.140
.150
.160
TIME FROM EXPLOSION OF PRIMER - SEC.

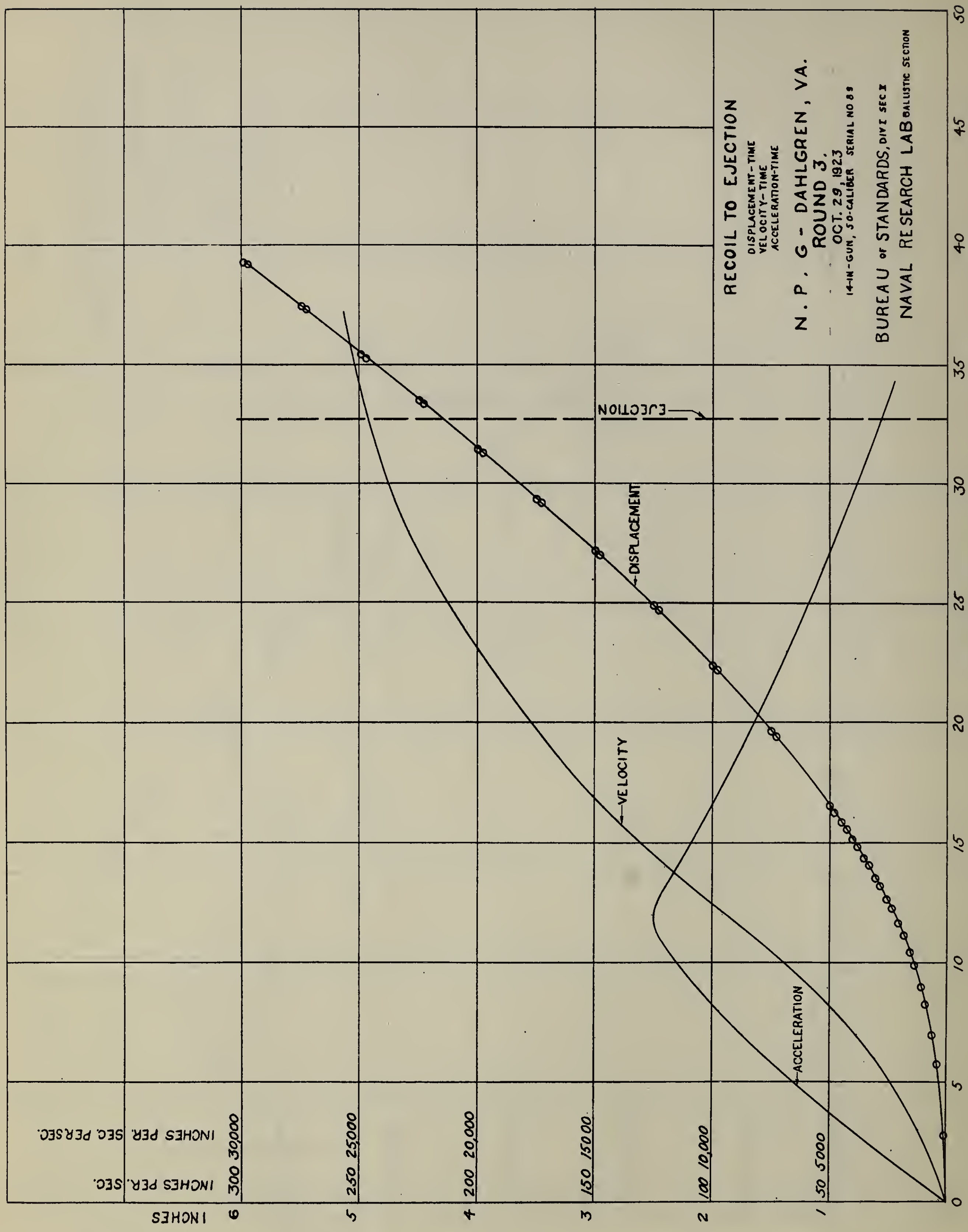
FIG. 12



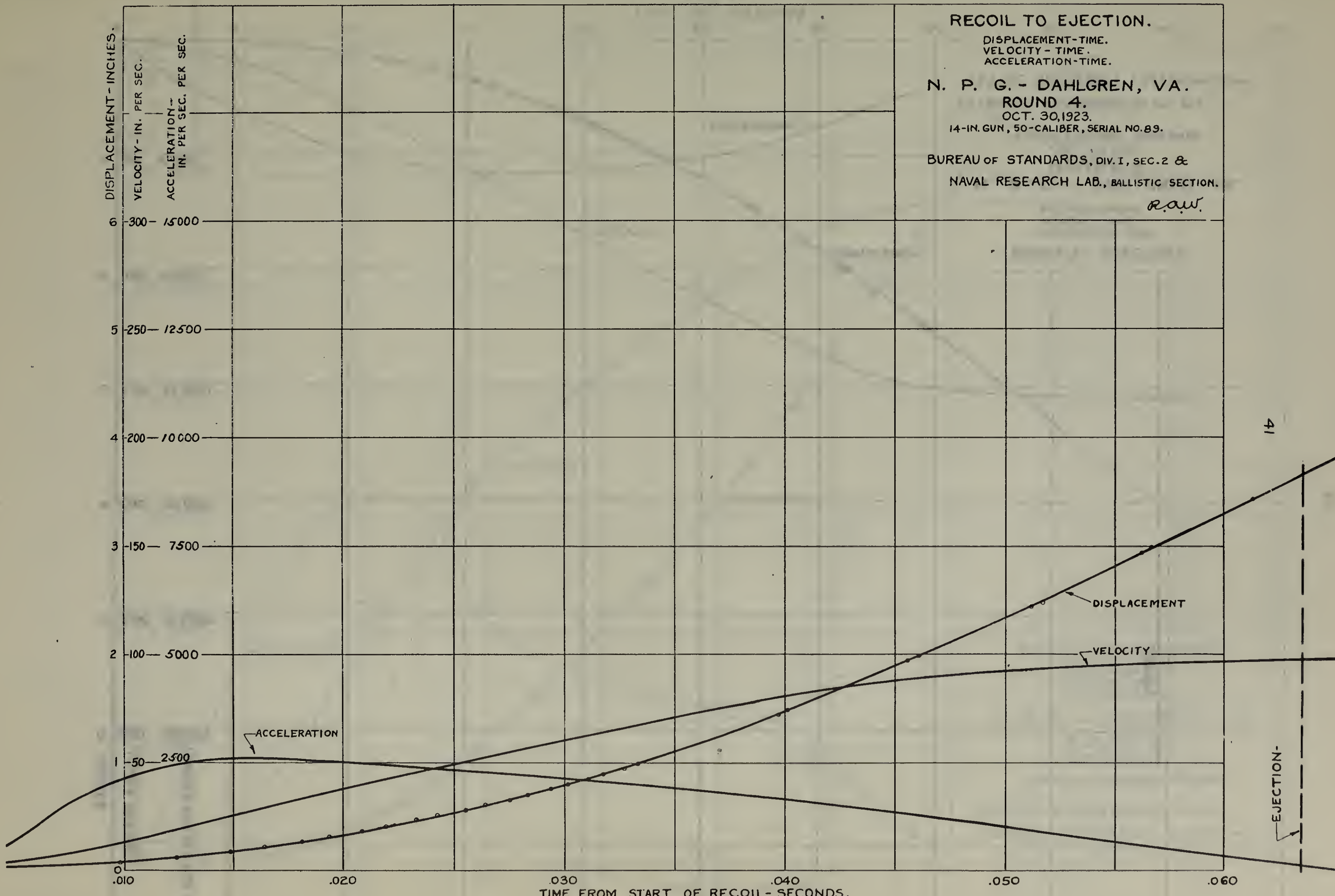
TIME-MILLISECONDS
FIG. 13



TIME-MILLISECONDS
FIG. 14



TIME—MILLISECONDS
FIG. 15


RECOIL TO EJECTION.
DISPLACEMENT-TIME.
VELOCITY - TIME.
ACCELERATION-TIME.
N. P. G. - DAHLGREN, VA.
ROUND 4.
OCT. 30,1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
BUREAU OF STANDARDS, DIV. I, SEC. 2 &
NAVAL RESEARCH LAB., BALLISTIC SECTION.
R.A.W.
DISPLACEMENT - INCHES.
VELOCITY - IN. PER SEC.
ACCELERATION - IN. PER SEC. PER SEC.
6 300 15000
5 250 12500
4 200 10000
3 150 7500
2 100 5000
1 50 2500
0
.010
.020
.030
.040
.050
.060
DISPLACEMENT
VELOCITY
ACCELERATION
EJECTION-
TIME FROM START OF RECOIL - SECONDS.
FIG. 16

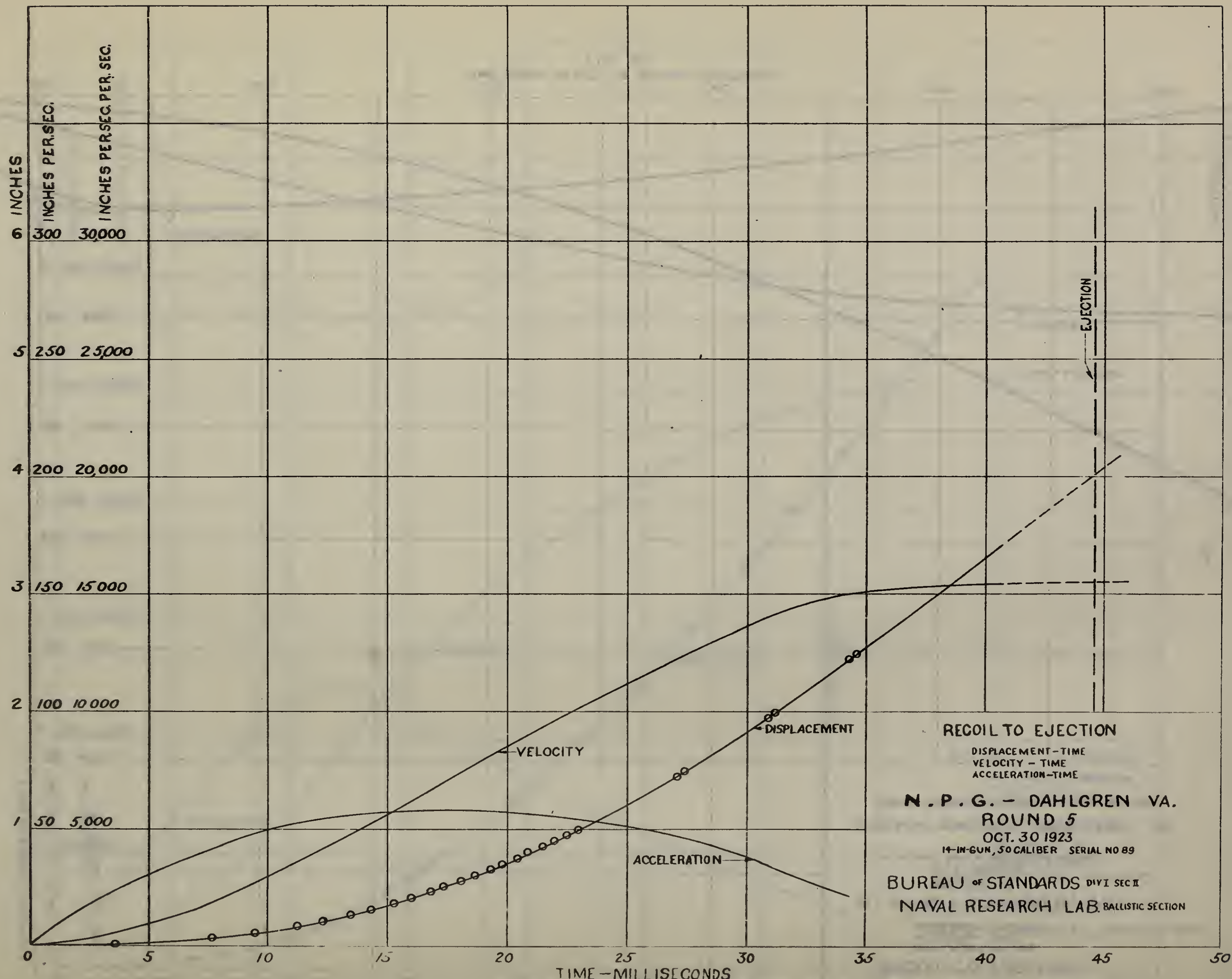

INCHES
INCHES PER SEC.
INCHES PER SEC. PER. SEC.
6 300 30,000
5 250 25,000
4 200 20,000
3 150 15000
2 100 10000
1 50 5,000
0
5
10
15
20
25
30
35
40
45
50
TIME—MILLISECONDS
EJECTION
VELOCITY
DISPLACEMENT
ACCELERATION
RECOIL TO EJECTION
DISPLACEMENT-TIME
VELOCITY - TIME
ACCELERATION-TIME
N. P. G. - DAHLGREN VA.
ROUND 5
OCT. 30 1923
14-IN-GUN, 50 CALIBER SERIAL NO 89
BUREAU of STANDARDS DIV I SEC II
NAVAL RESEARCH LAB. BALLISTIC SECTION

FIG. 17

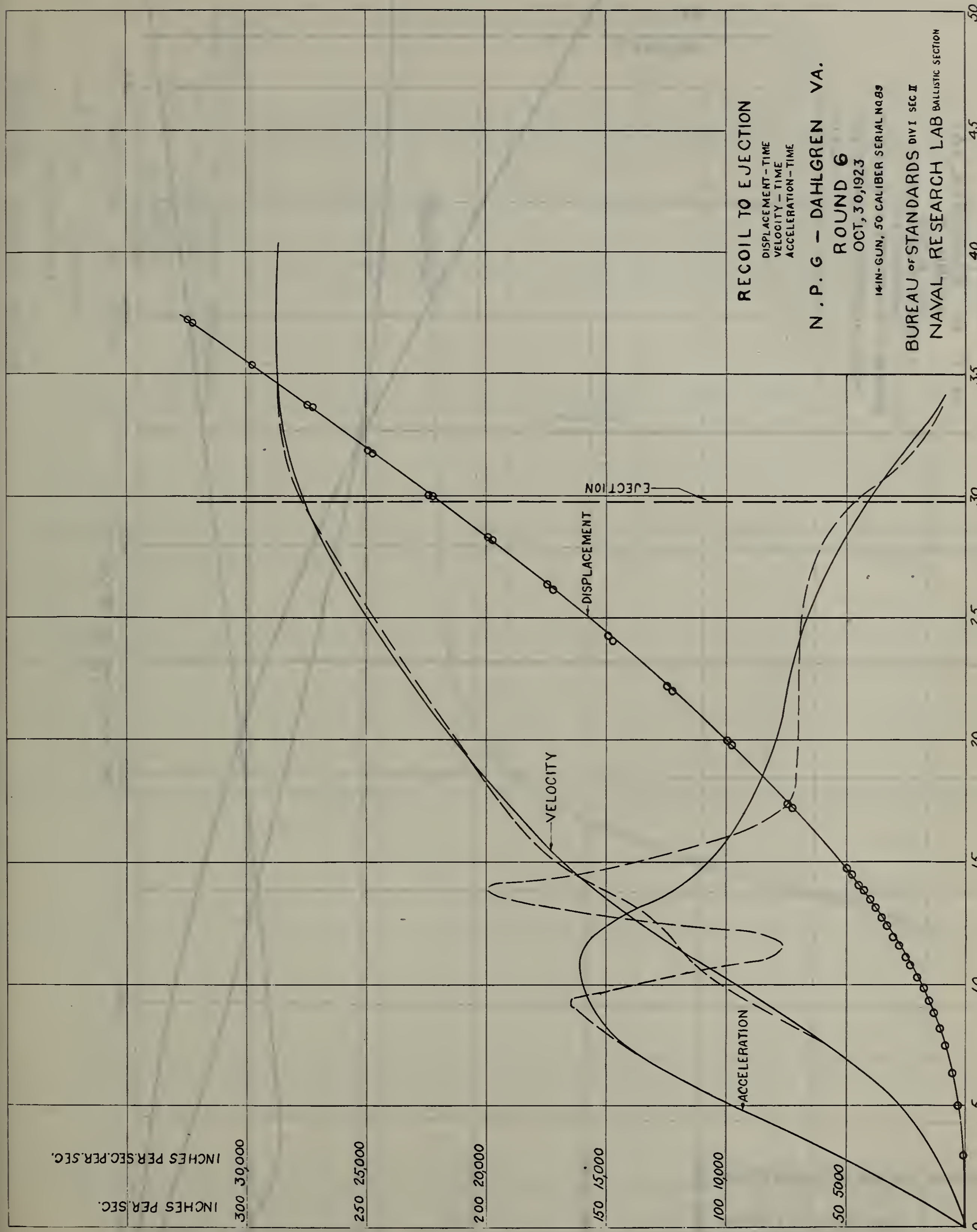
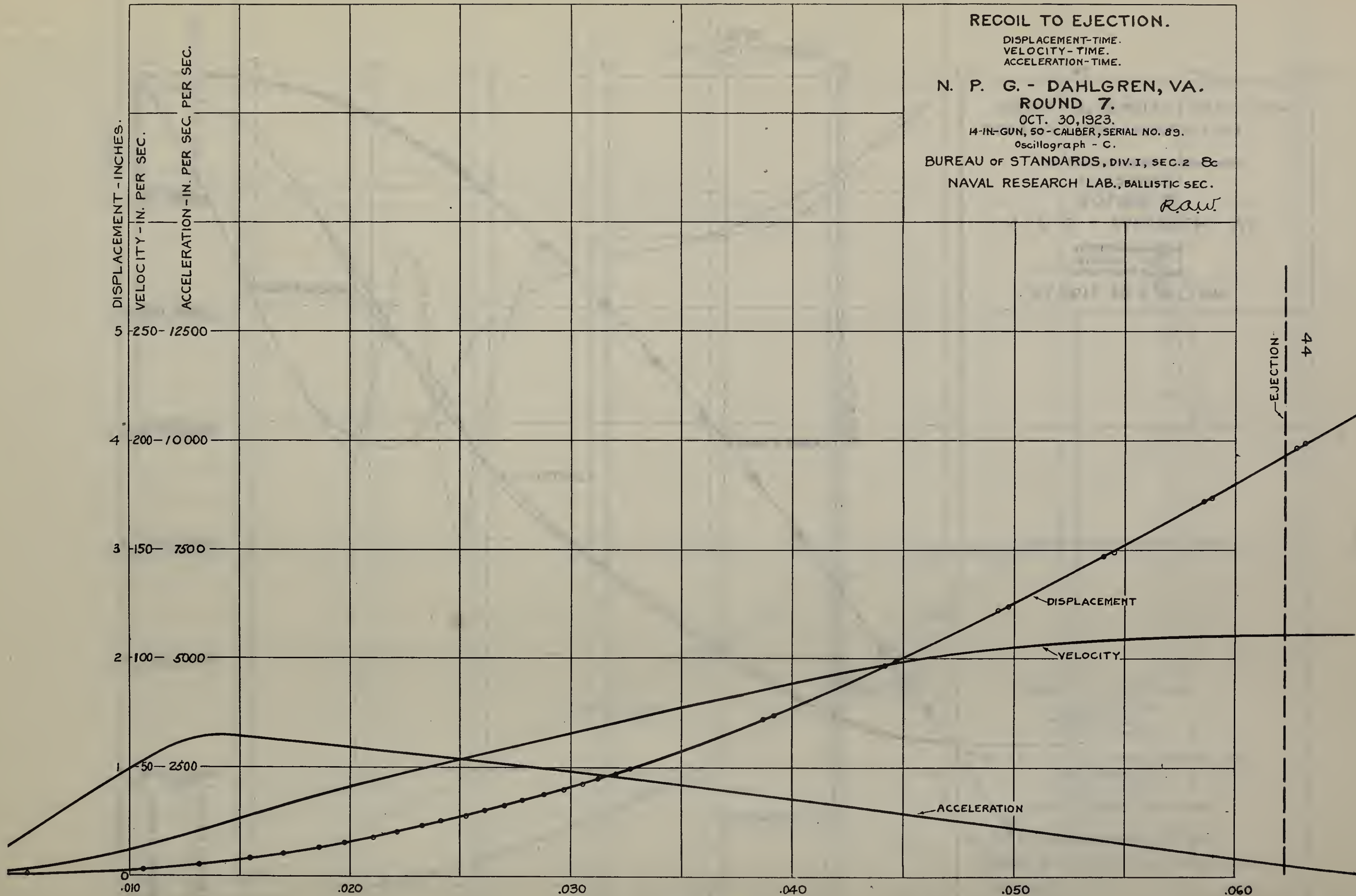
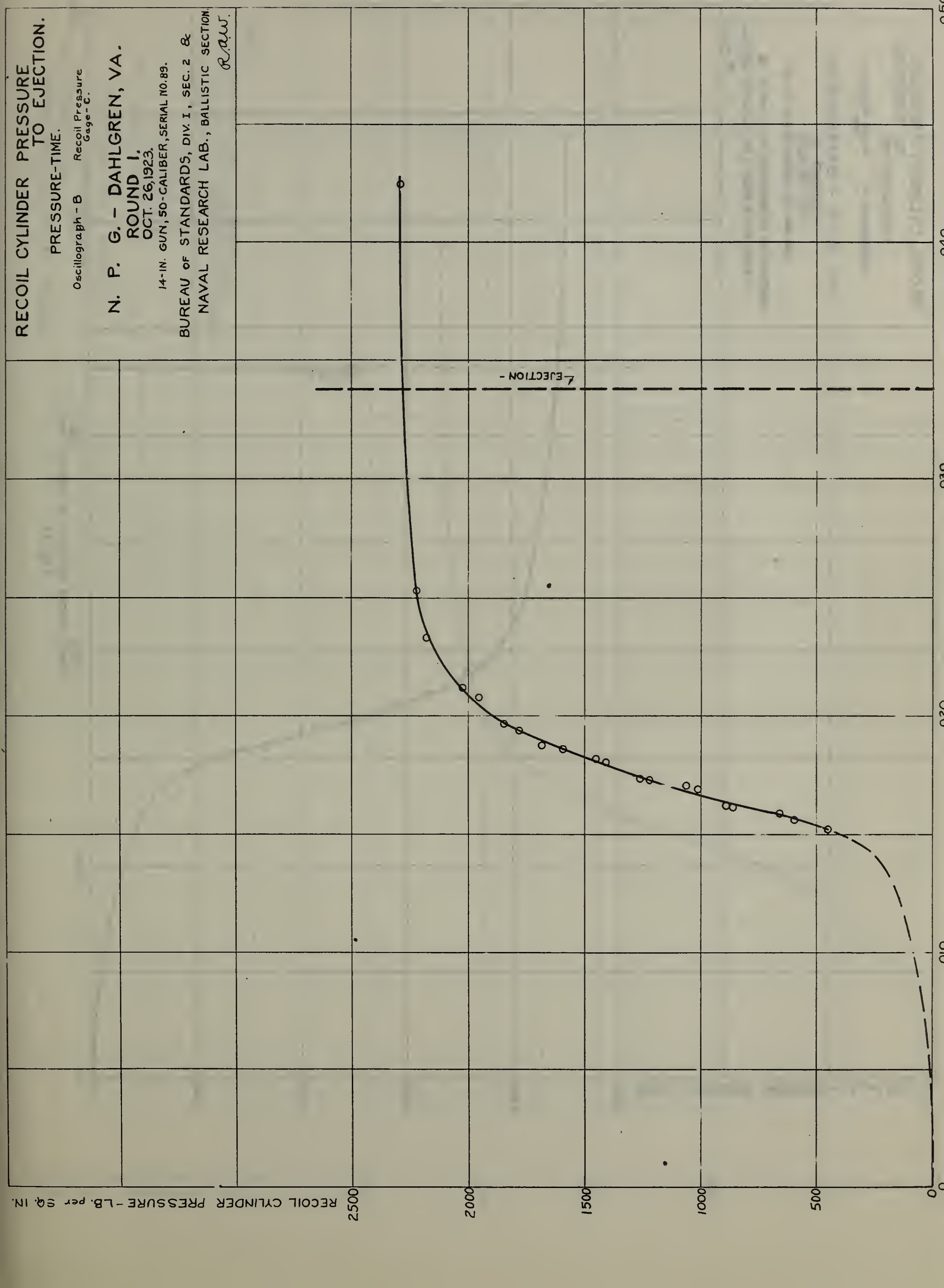

RECOIL TO EJECTION
DISPLACEMENT-TIME
VELOCITY-TIME
ACCELERATION-TIME
N. P. G - DAHLGREN VA.
ROUND 6
OCT, 30, 1923
14-IN-GUN, 50 CALIBER SERIAL No.89
BUREAU of STANDARDS DIV I SEC II
NAVAL RESEARCH LAB BALLISTIC SECTION
EJECTION
DISPLACEMENT
VELOCITY
ACCELERATION
INCHES
INCHES PER SEC.
INCHES PER SEC. PER SEC.
6 300 30,000
5 250 25,000
4 200 20,000
3 150 15,000
2 100 10,000
1 50 5000
0
0 5 10 15 20 25 30 35 40 45 50
TIME-MILLISECONDS

FIG.18



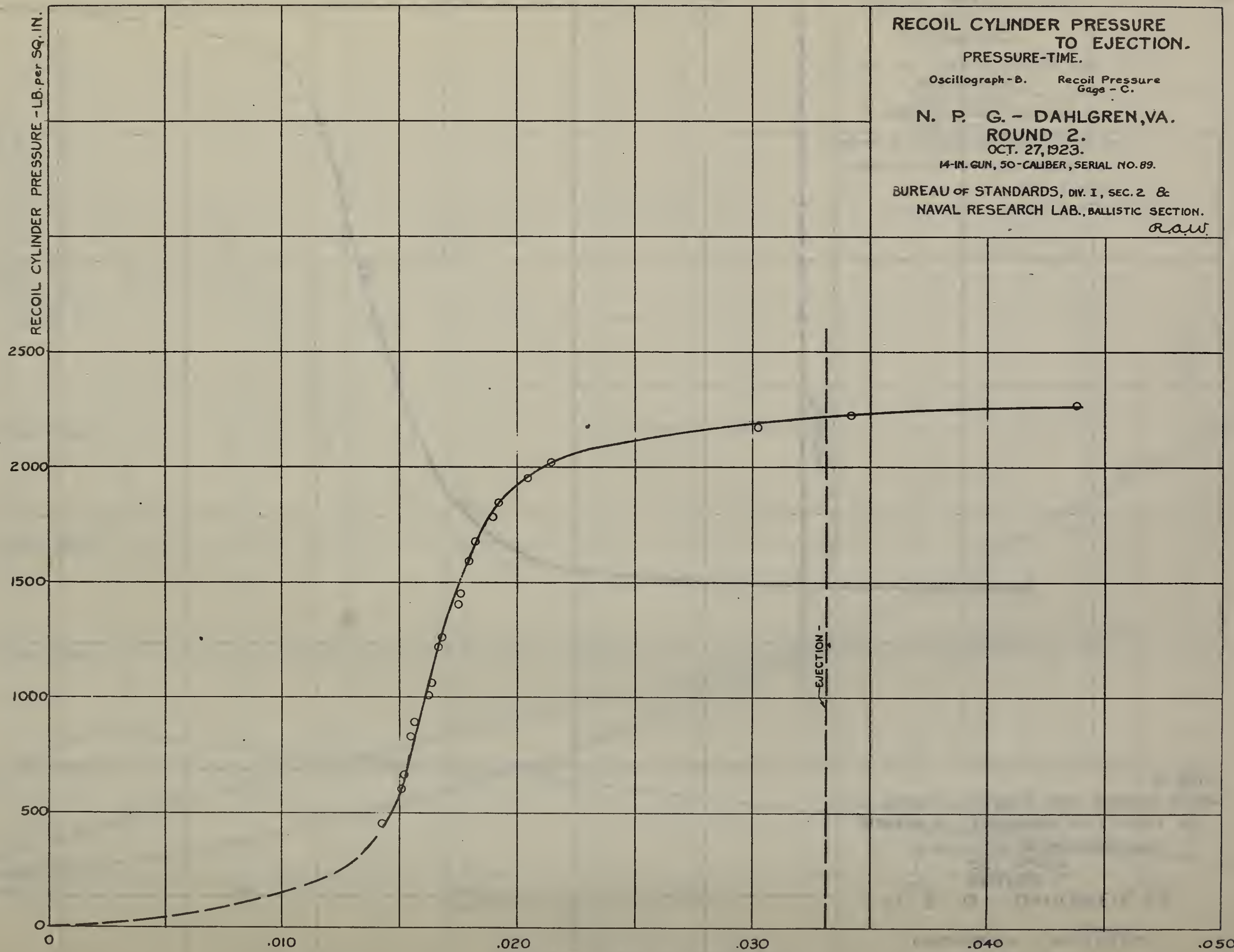
TIME FROM START OF RECOIL - SEC.

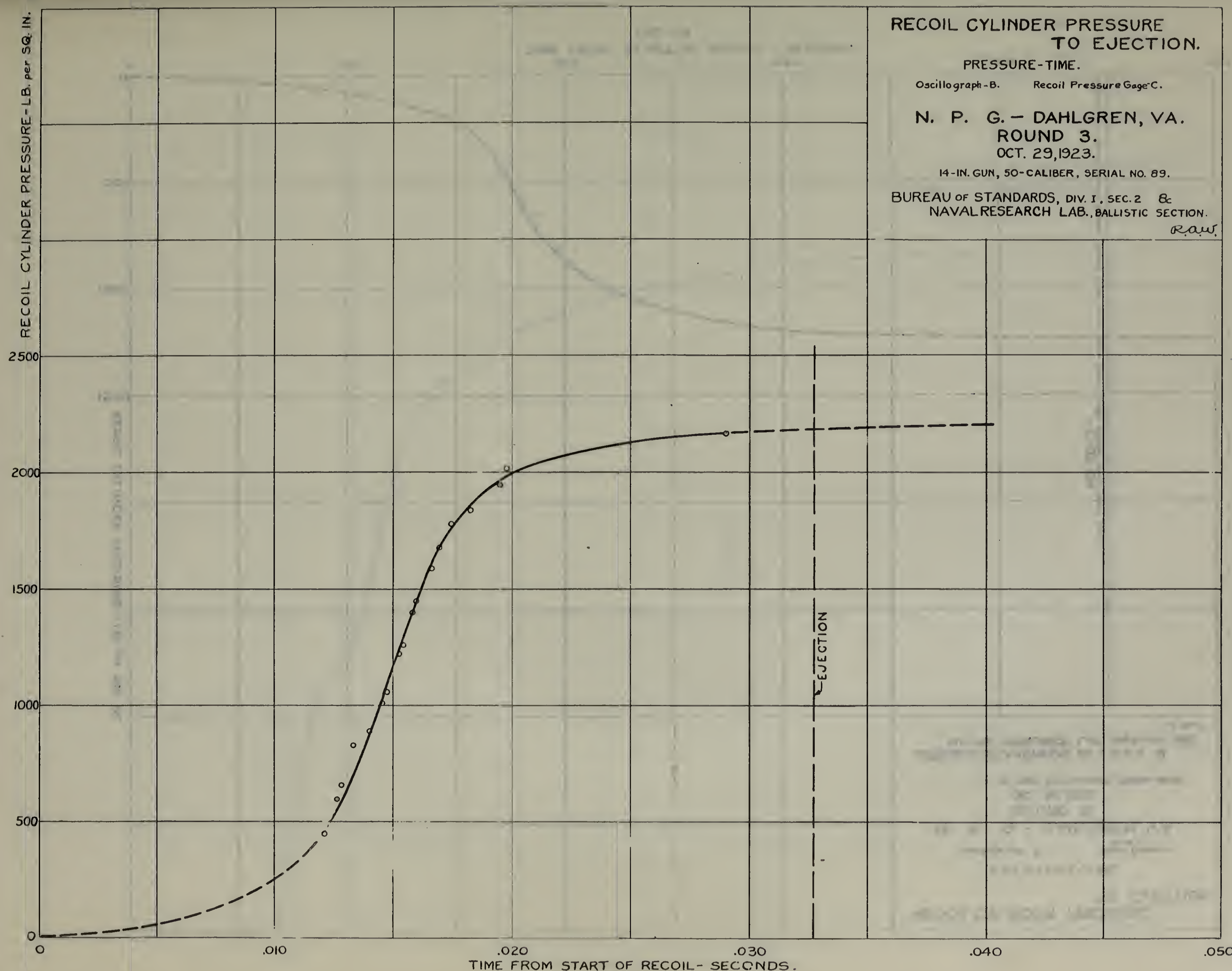
FIG. 19



RECOIL CYLINDER PRESSURE TO EJECTION.
PRESSURE-TIME.
Oscillograph - B
Recoil Pressure Gage - C.
N. P. G. - DAHLGREN, VA.
ROUND I.
OCT. 26, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
BUREAU OF STANDARDS, DIV. I, SEC. 2 &
NAVAL RESEARCH LAB., BALLISTIC SECTION.
R.A.W.
EJECTION
RECOIL CYLINDER PRESSURE - LB. per SQ. IN.
2500
2000
1500
1000
500
0
0
.010
.020
.030
.040
.050
TIME FROM START OF RECOIL - SEC.

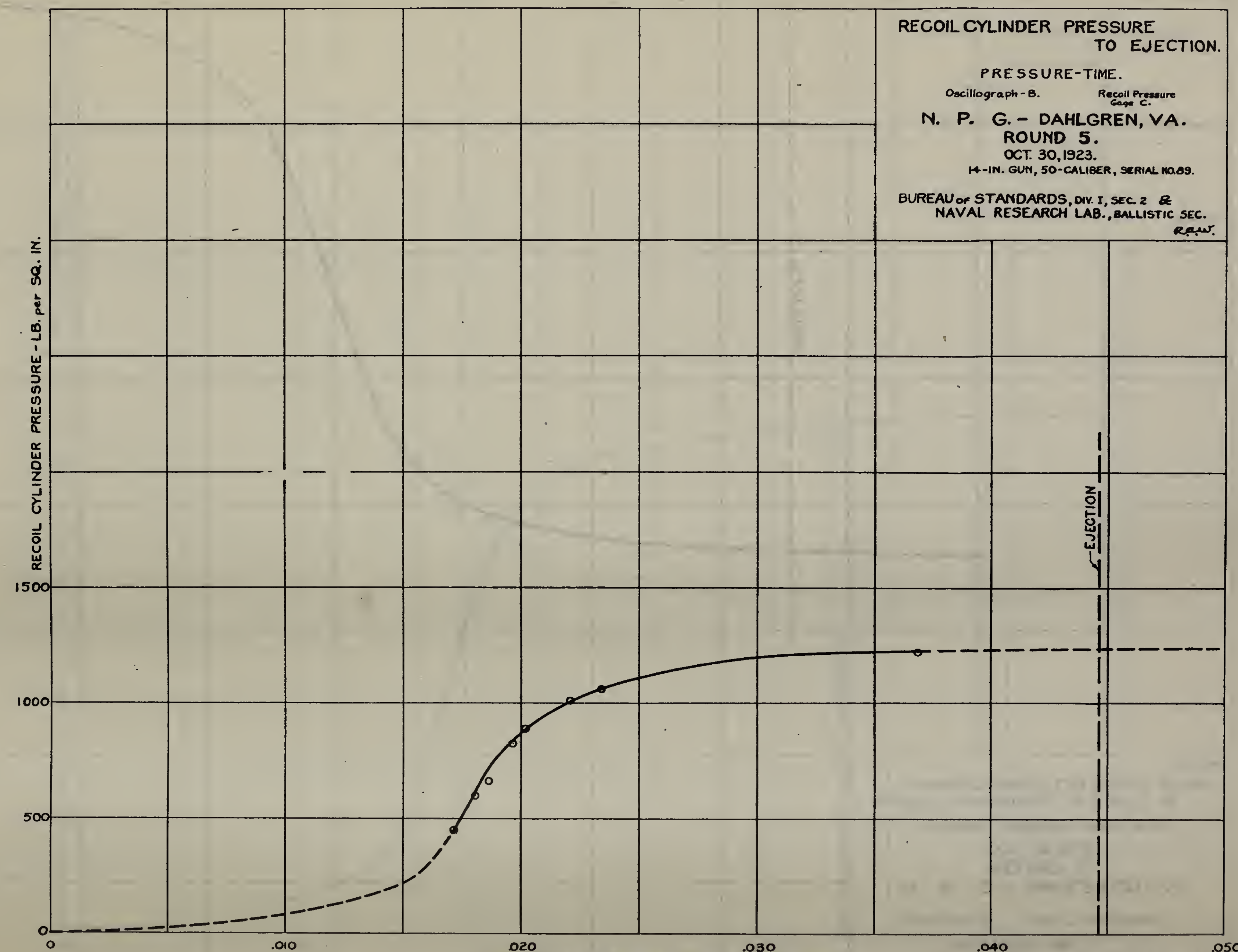
FIG. 20

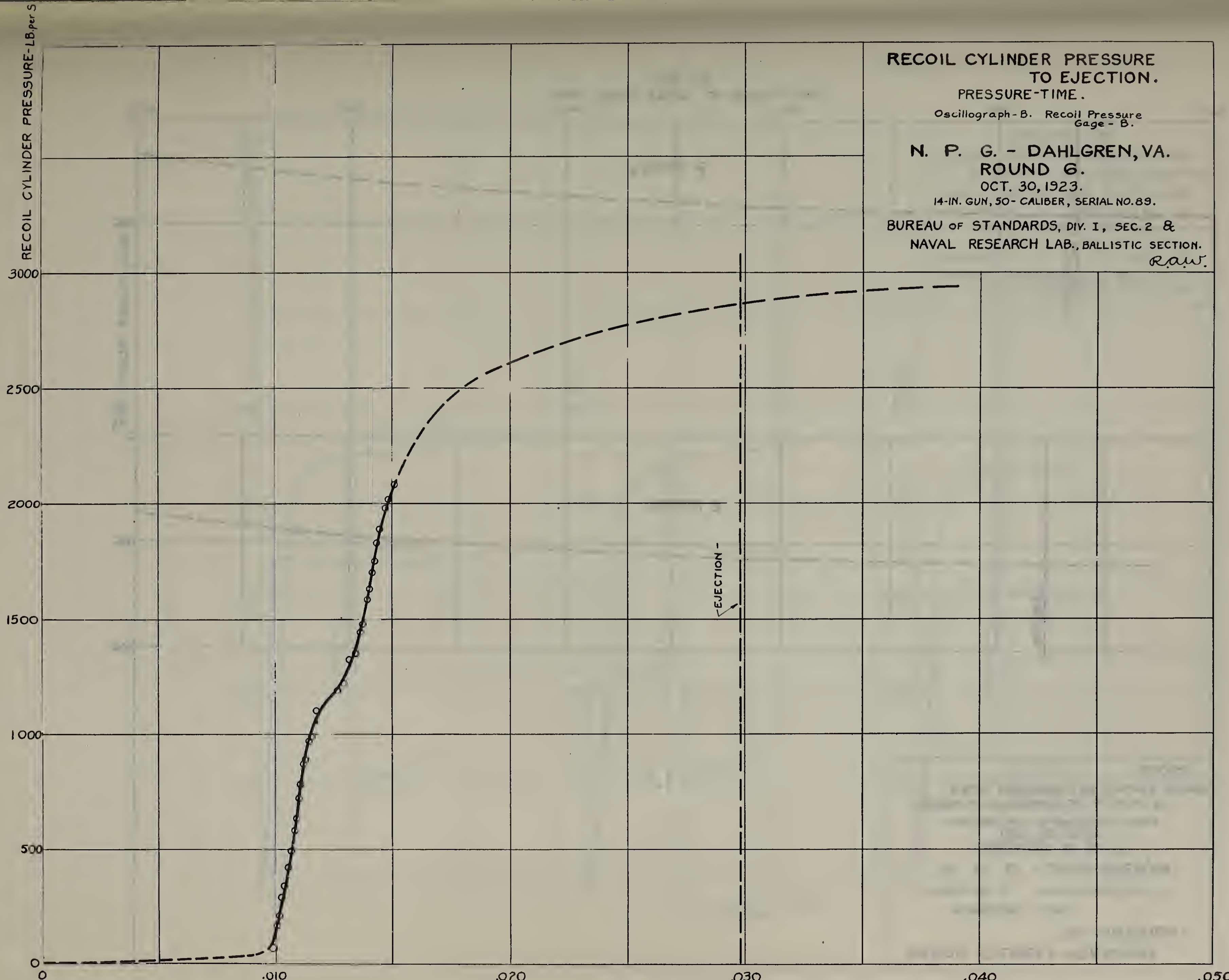

RECOIL CYLINDER PRESSURE TO EJECTION.
PRESSURE-TIME.
Oscillograph - B.
Recoil Pressure Gage - C.
N. P. G. - DAHLGREN, VA.
ROUND 2.
OCT. 27, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
BUREAU OF STANDARDS, DIV. I, SEC. 2 &
NAVAL RESEARCH LAB., BALLISTIC SECTION.
R.a.W.
RECOIL CYLINDER PRESSURE - LB. per SQ. IN.
2500
2000
1500
1000
500
0
0
.010
.020
.030
.040
.050
EJECTION
TIME FROM START OF RECOIL - SEC.
FIG. 21

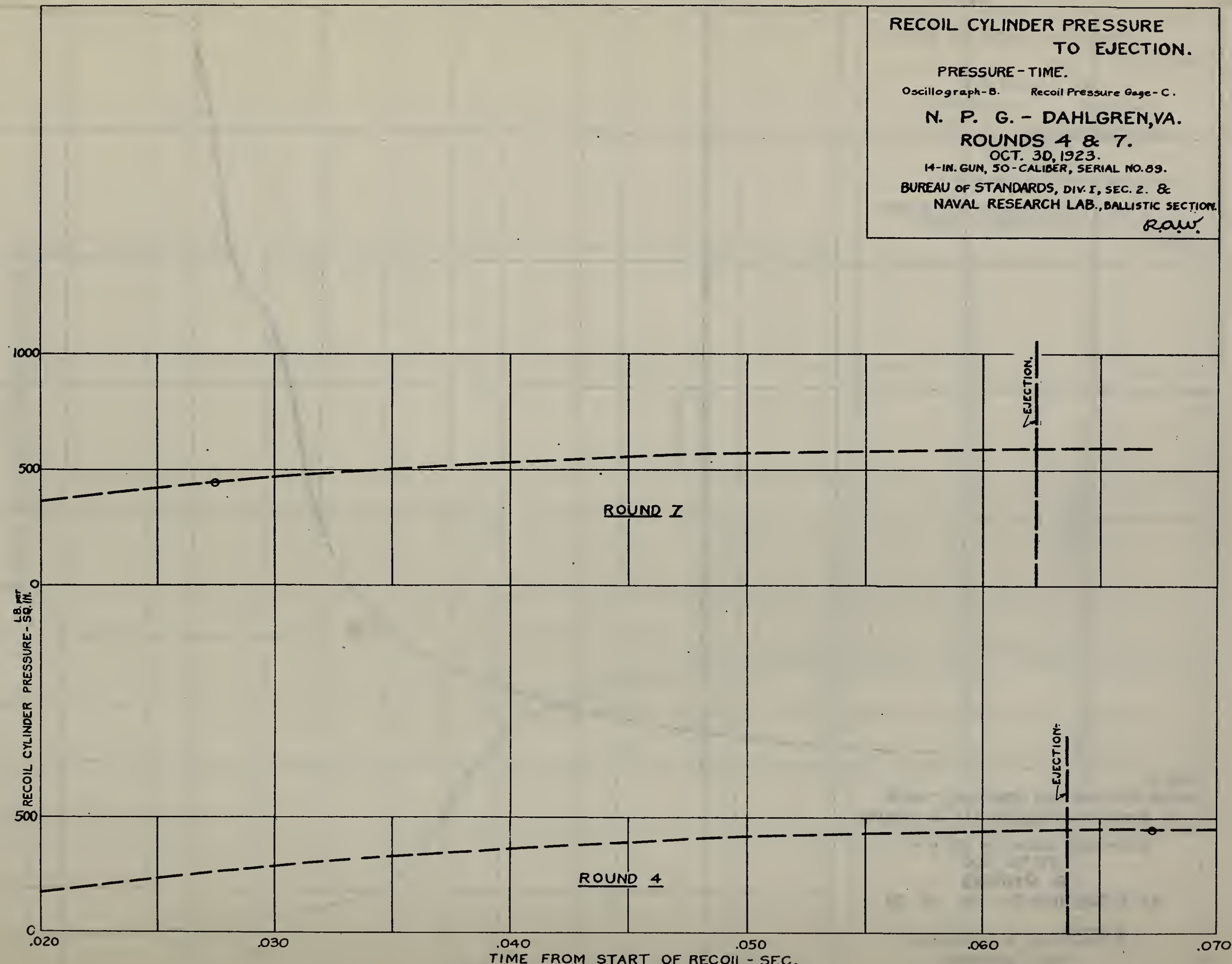


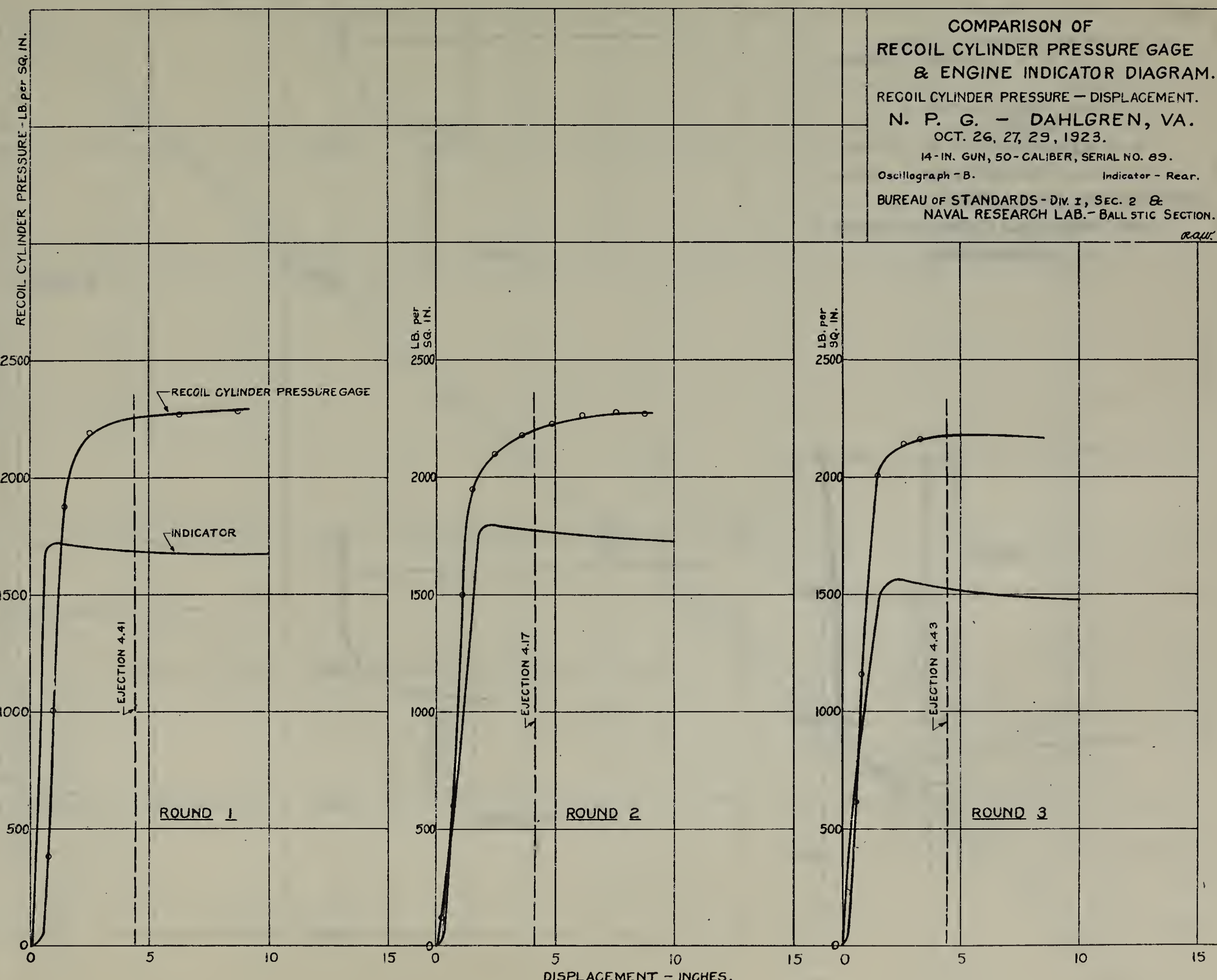
RECOIL CYLINDER PRESSURE
TO EJECTION.
PRESSURE-TIME.
Oscillograph-B. Recoil Pressure Gage C.
N. P. G. – DAHLGREN, VA.
ROUND 3.
OCT. 29, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
BUREAU OF STANDARDS, DIV. I, SEC. 2 &
NAVAL RESEARCH LAB., BALLISTIC SECTION.
R.A.W.
EJECTION
RECOIL CYLINDER PRESSURE-LB. per SQ. IN.
2500
2000
1500
1000
500
0
0
.010
.020
.030
.040
.050
TIME FROM START OF RECOIL- SECONDS.

FIG. 22


RECOIL CYLINDER PRESSURE TO EJECTION.
PRESSURE-TIME.
Oscillograph - B.
Recoil Pressure Gage C.
N. P. G. - DAHLGREN, VA.
ROUND 5.
OCT. 30, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
BUREAU of STANDARDS, DIV. I, SEC. 2 & NAVAL RESEARCH LAB., BALLISTIC SEC.
R.A.W.
EJECTION
RECOIL CYLINDER PRESSURE - LB. per SQ. IN.
0
500
1000
1500
0
.010
.020
.030
.040
.050
TIME FROM START OF RECOIL - SECONDS.
FIG. 23


RECOIL CYLINDER PRESSURE TO EJECTION.
PRESSURE-TIME.
Oscillograph-B. Recoil Pressure Gage-B.
N. P. G. - DAHLGREN, VA.
ROUND 6.
OCT. 30, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
BUREAU of STANDARDS, DIV. I, SEC. 2 & NAVAL RESEARCH LAB., BALLISTIC SECTION.
R.A.W.
RECOIL CYLINDER PRESSURE-LB. per S
3000
2500
2000
1500
1000
500
0
0
.010
.020
.030
.040
.050
EJECTION
TIME FROM START OF RECOIL-SECONDS.
FIG. 24


RECOIL CYLINDER PRESSURE TO EJECTION.
PRESSURE - TIME.
Oscillograph - B. Recoil Pressure Gage - C.
N. P. G. - DAHLGREN, VA.
ROUNDS 4 & 7.
OCT. 30, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
BUREAU OF STANDARDS, DIV. I, SEC. 2. & NAVAL RESEARCH LAB., BALLISTIC SECTION.
R.A.W.
EJECTION.
EJECTION.
ROUND 7
ROUND 4
RECOIL CYLINDER PRESSURE - LB. PER SQ. IN.
1000
500
0
500
0
.020
.030
.040
.050
.060
.070
TIME FROM START OF RECOIL - SEC.
FIG. 25



COMPARISON OF
RECOIL CYLINDER PRESSURE GAGE
& ENGINE INDICATOR DIAGRAM.
RECOIL CYLINDER PRESSURE — DISPLACEMENT.
N. P. G. — DAHLGREN, VA.
OCT. 26, 27, 29, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
Oscillograph - B.
Indicator - Rear.
BUREAU OF STANDARDS - DIV. I, SEC. 2 &
NAVAL RESEARCH LAB. - BALLISTIC SECTION.
RECOIL CYLINDER PRESSURE - LB. per SQ. IN.
LB. per SQ. IN.
RECOIL CYLINDER PRESSURE GAGE
INDICATOR
EJECTION 4.41
EJECTION 4.17
EJECTION 4.43
ROUND 1
ROUND 2
ROUND 3
0
500
1000
1500
2000
2500
0
5
10
15
DISPLACEMENT - INCHES.

FIG. 26

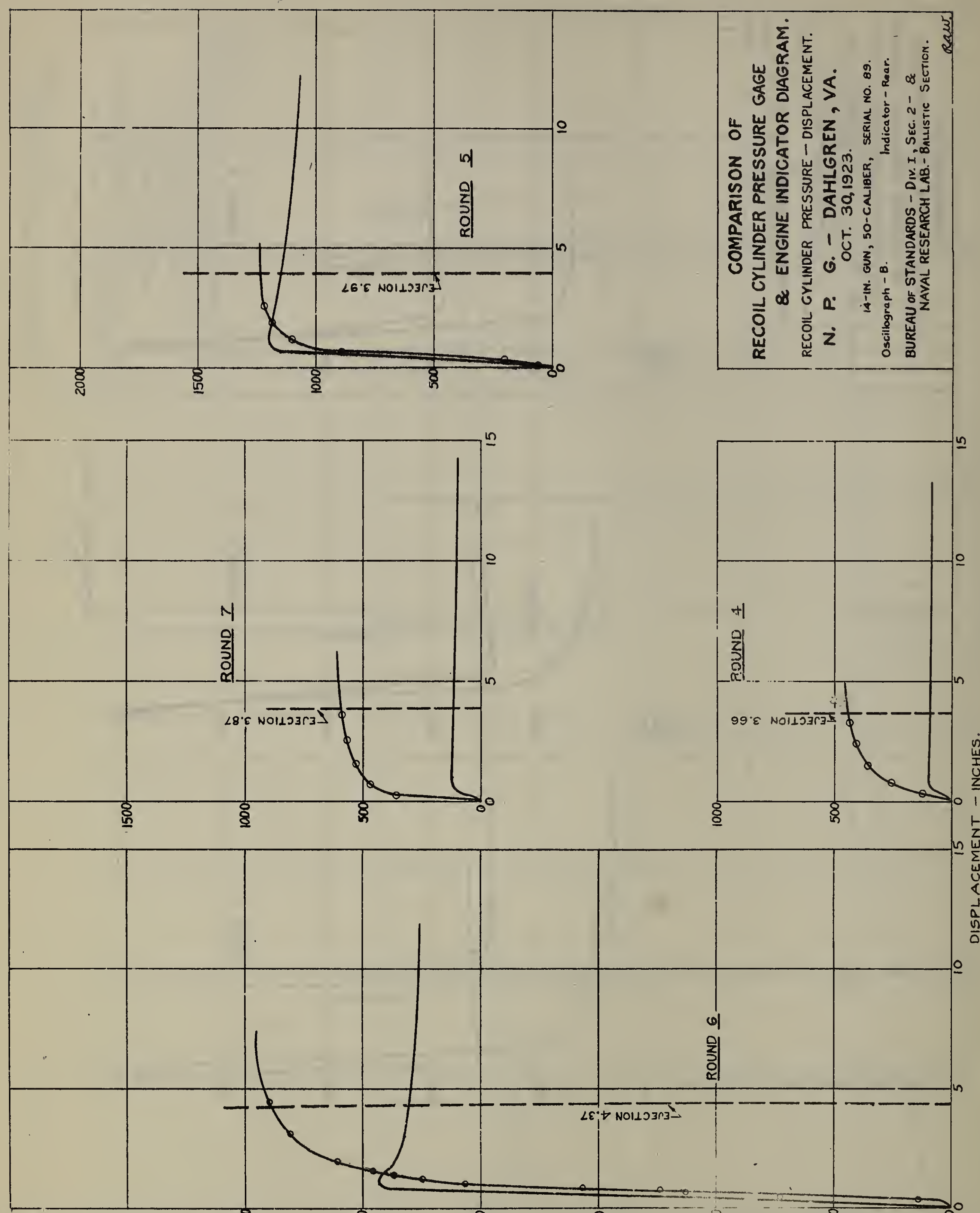
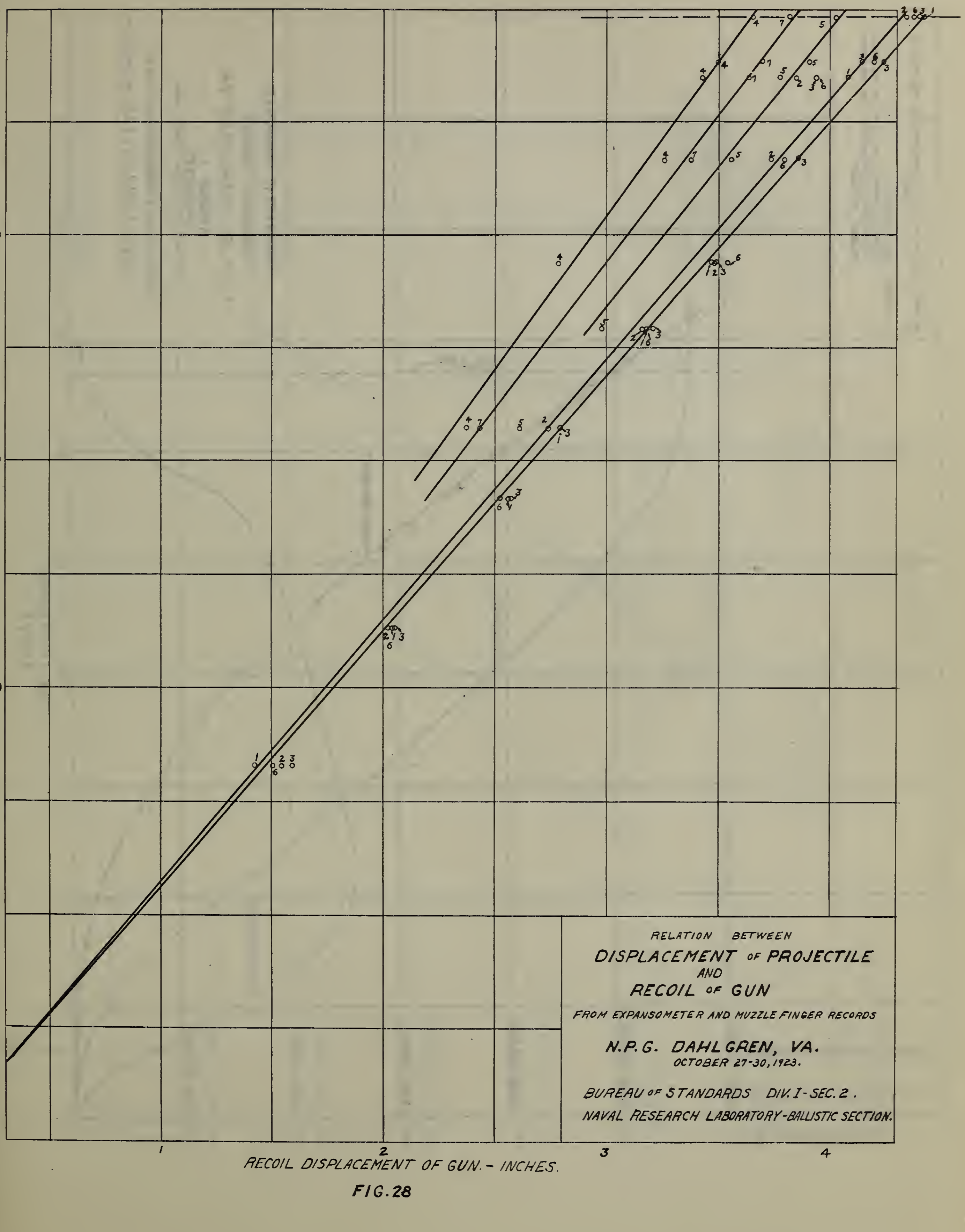
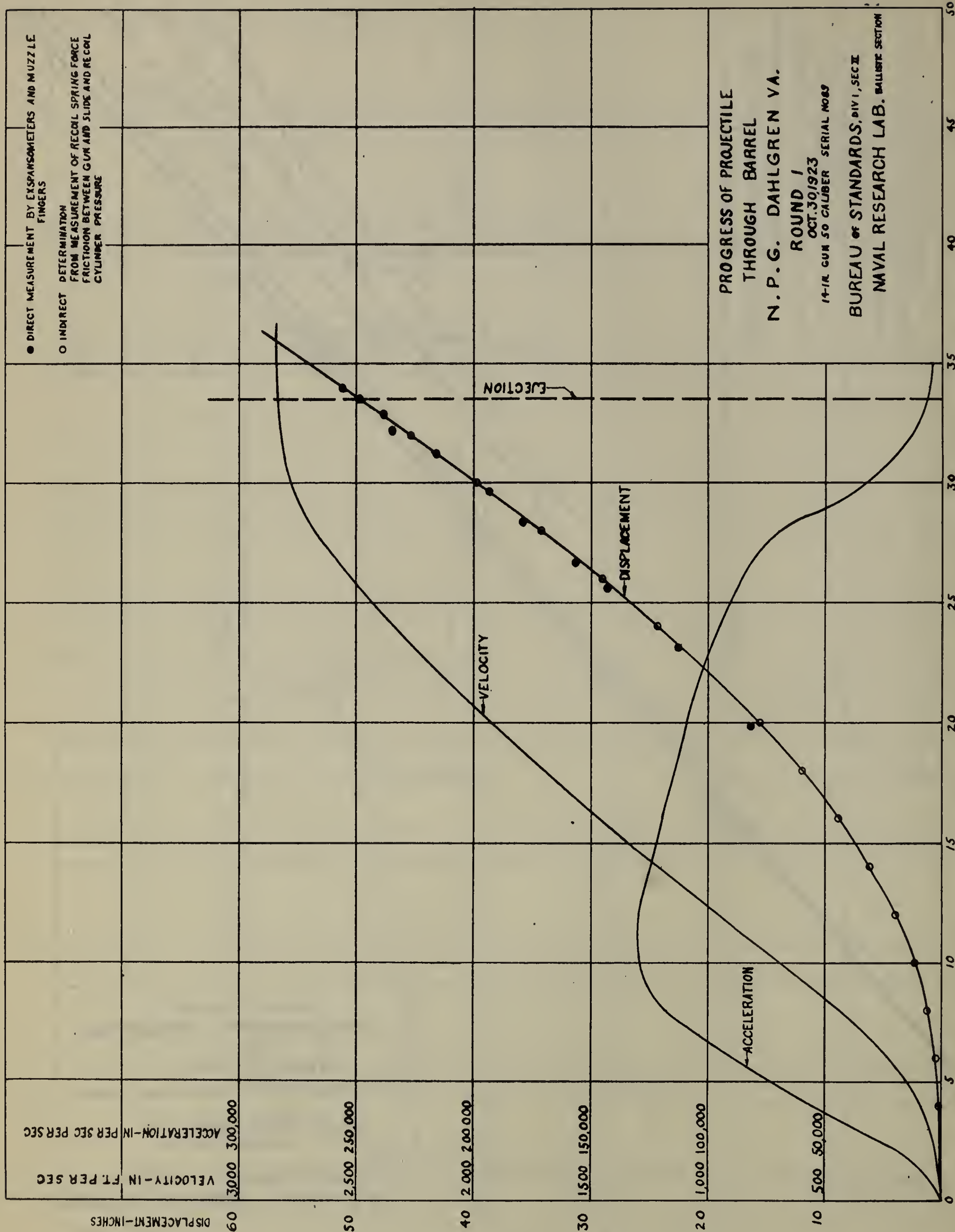

COMPARISON OF
RECOIL CYLINDER PRESSURE GAGE
& ENGINE INDICATOR DIAGRAM.
RECOIL CYLINDER PRESSURE – DISPLACEMENT.
N. P. G. – DAHLGREN, VA.
OCT. 30, 1923.
14-IN. GUN, 50-CALIBER, SERIAL NO. 89.
Oscillograph - B.
Indicator - Rear.
BUREAU OF STANDARDS – DIV. I, SEC. 2 – &
NAVAL RESEARCH LAB. - BALLISTIC SECTION.
ROUND 5
EJECTION 3.97
ROUND 7
EJECTION 3.87
ROUND 4
EJECTION 3.66
ROUND 6
EJECTION 4.37
REC. CYL. PRESSURE – LB. per SQ. IN.
DISPLACEMENT – INCHES.

FIG. 27



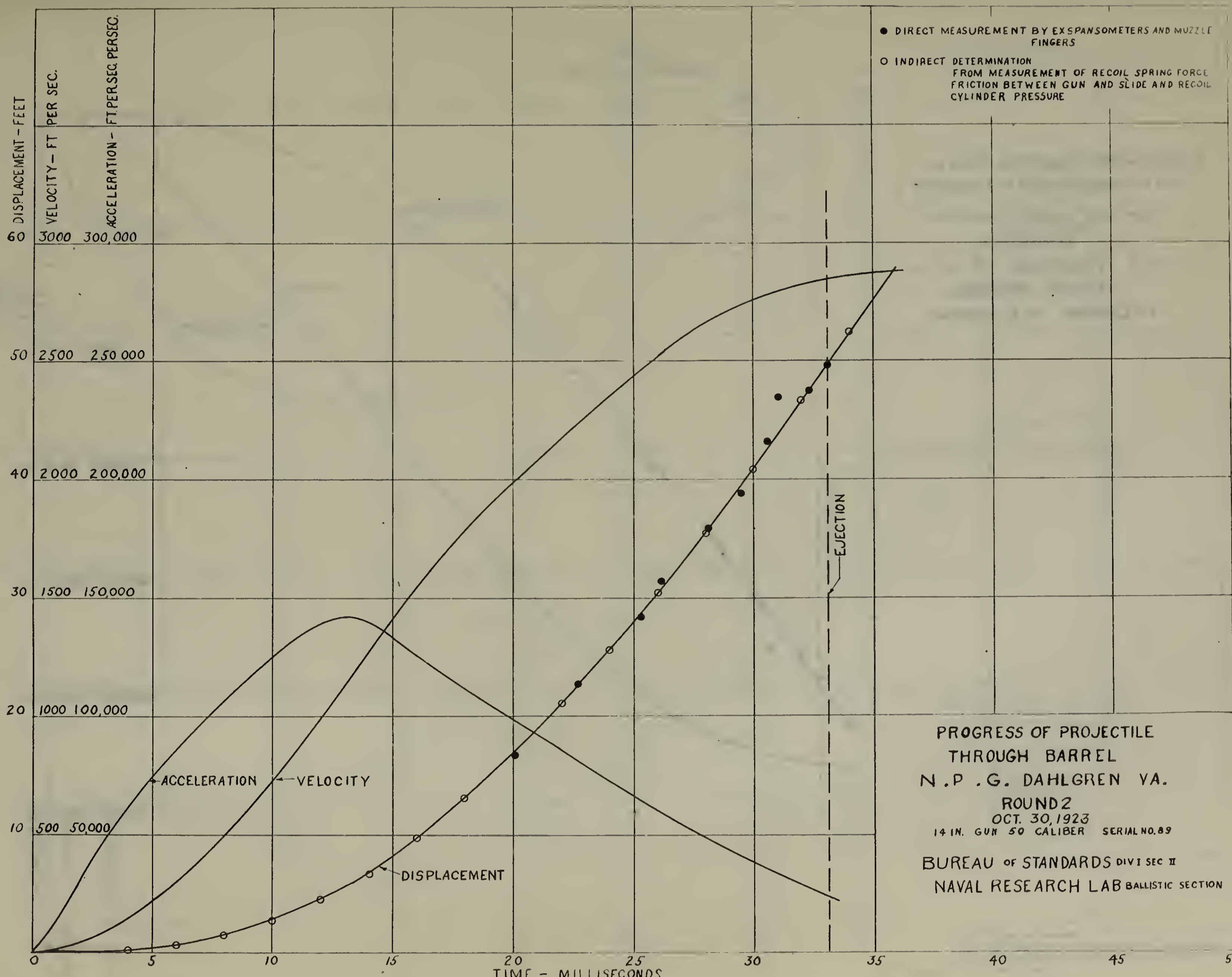
50
40
30
20
10
DISPLACEMENT OF SHELL IN GUN. - FEET.
1
2
3
4
RECOIL DISPLACEMENT OF GUN. - INCHES.
RELATION BETWEEN
DISPLACEMENT OF PROJECTILE
AND
RECOIL OF GUN
FROM EXPANSOMETER AND MUZZLE FINGER RECORDS
N.P.G. DAHLGREN, VA.
OCTOBER 27-30, 1923.
BUREAU OF STANDARDS DIV. I - SEC. 2.
NAVAL RESEARCH LABORATORY - BALLISTIC SECTION.

FIG. 28



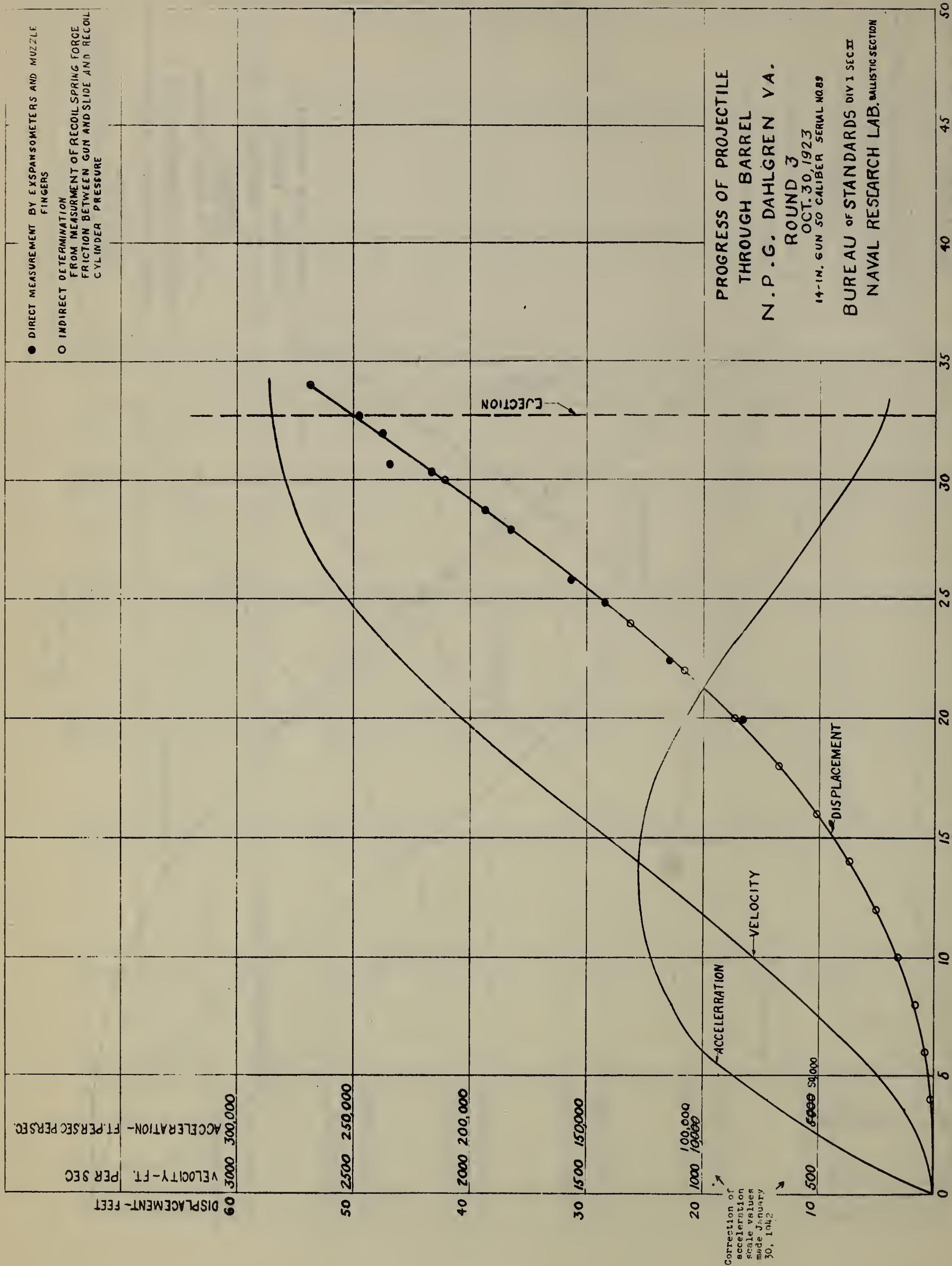
TIME – MILLI SECONDS

FIG.29

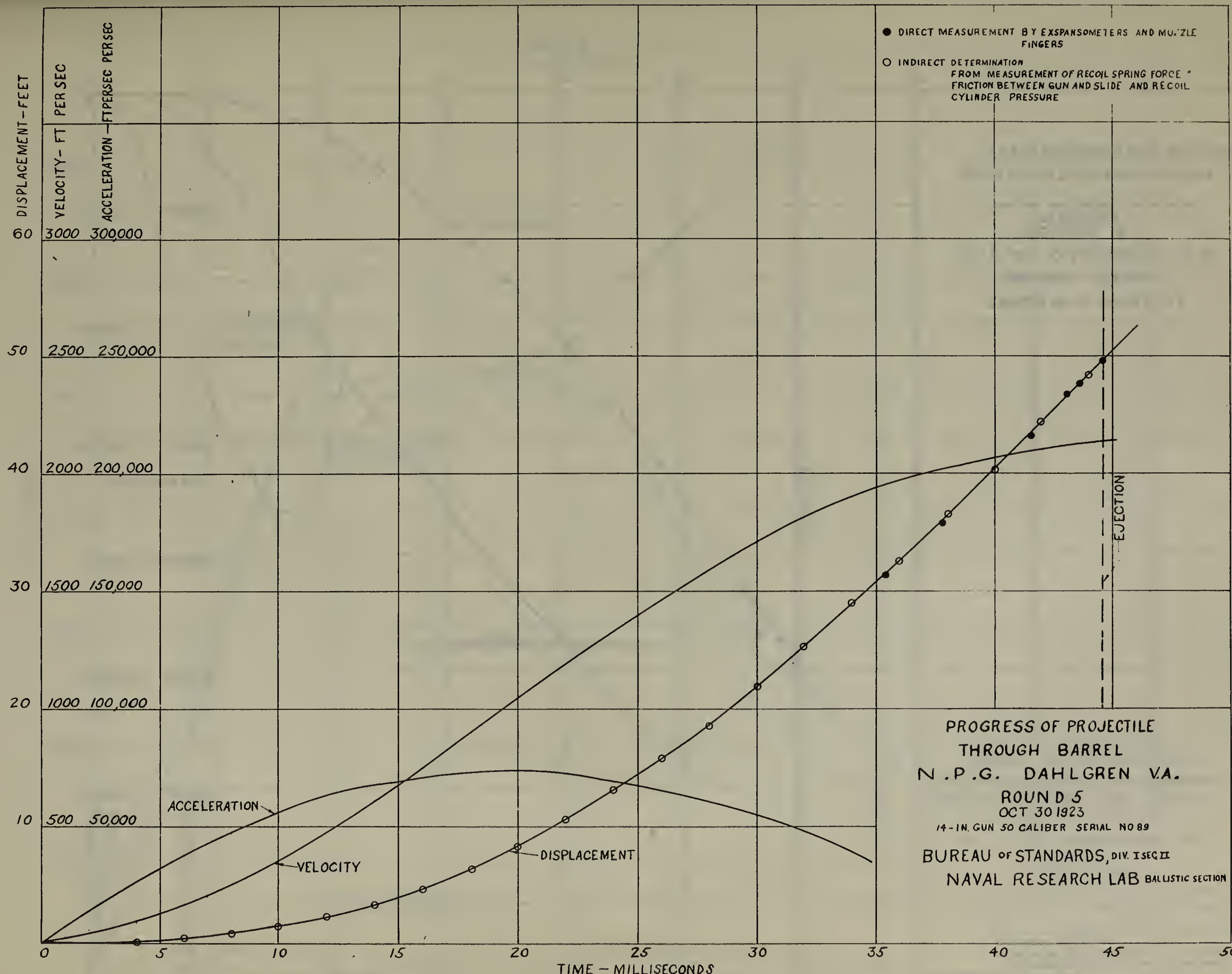


● DIRECT MEASUREMENT BY EXSPANSOMETERS AND MUZZLE FINGERS
O INDIRECT DETERMINATION FROM MEASUREMENT OF RECOIL SPRING FORCE FRICTION BETWEEN GUN AND SLIDE AND RECOIL CYLINDER PRESSURE
DISPLACEMENT - FEET
VELOCITY - FT PER SEC.
ACCELERATION - FT.PER.SEC. PER.SEC.
60 3000 300,000
50 2500 250 000
40 2000 200,000
30 1500 150,000
20 1000 100,000
10 500 50,000
ACCELERATION
VELOCITY
DISPLACEMENT
EJECTION
PROGRESS OF PROJECTILE
THROUGH BARREL
N.P.G. DAHLGREN VA.
ROUND 2
OCT. 30, 1923
14 IN. GUN 50 CALIBER SERIAL NO. 89
BUREAU OF STANDARDS DIV I SEC II
NAVAL RESEARCH LAB BALLISTIC SECTION
0 5 10 15 20 25 30 35 40 45 50
TIME - MILLISECONDS

FIG. 30



TIME - MILLISECONDS
FIG. 31



DISPLACEMENT-FEET
VELOCITY- FT PER SEC
ACCELERATION -FTPERSEC PERSEC
60 3000 300,000
50 2500 250,000
40 2000 200,000
30 1500 150,000
20 1000 100,000
10 500 50,000
0 5 10 15 20 25 30 35 40 45 50
TIME - MILLISECONDS
ACCELERATION
VELOCITY
DISPLACEMENT
EJECTION
● DIRECT MEASUREMENT BY EXSPANSOMETERS AND MUZZLE FINGERS
○ INDIRECT DETERMINATION FROM MEASUREMENT OF RECOIL SPRING FORCE + FRICTION BETWEEN GUN AND SLIDE AND RECOIL CYLINDER PRESSURE
PROGRESS OF PROJECTILE
THROUGH BARREL
N.P.G. DAHLGREN VA.
ROUND 5
OCT 30 1923
14-IN. GUN 50 CALIBER SERIAL NO 89
BUREAU OF STANDARDS, DIV. I SEC II
NAVAL RESEARCH LAB BALLISTIC SECTION

FIG. 32

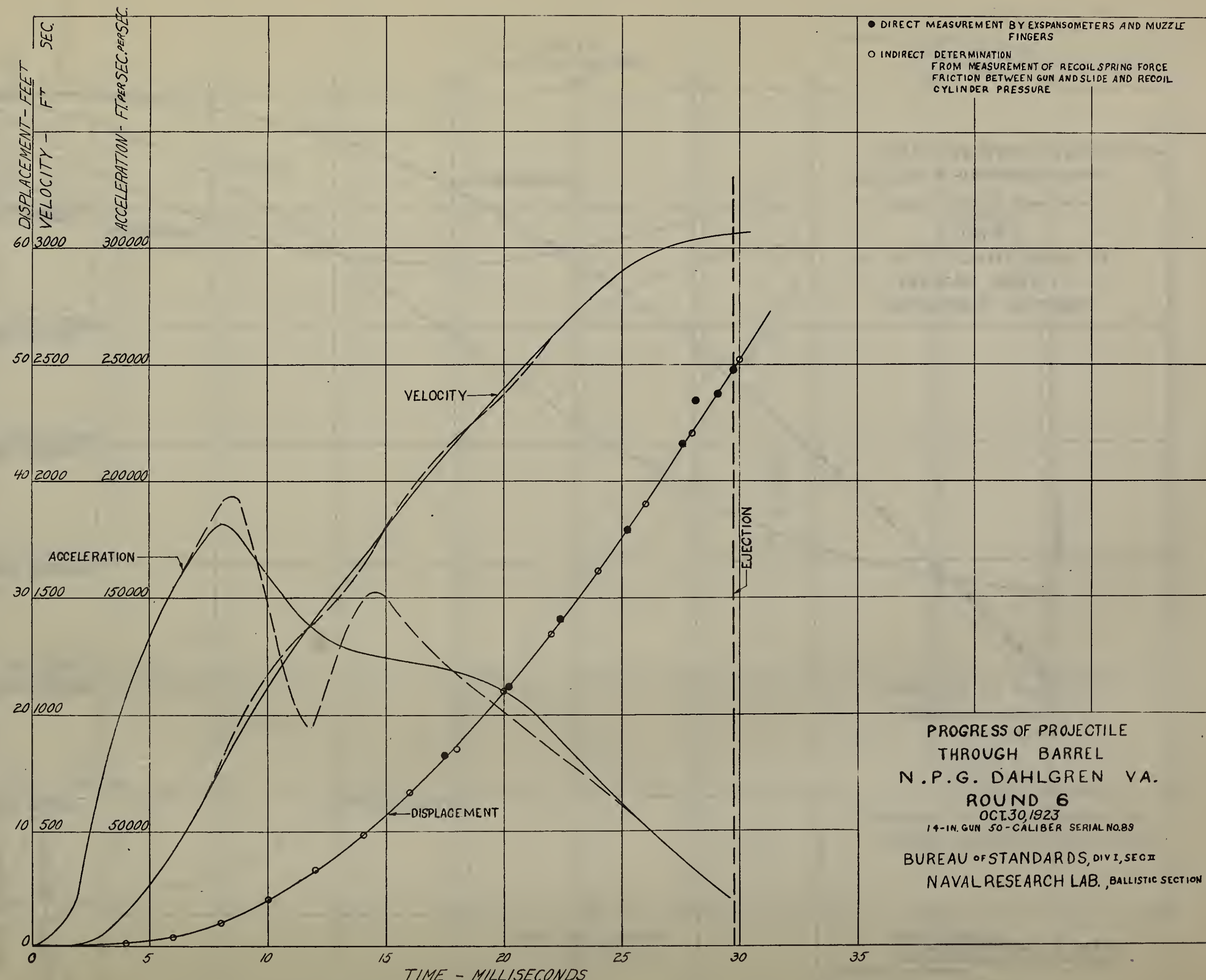
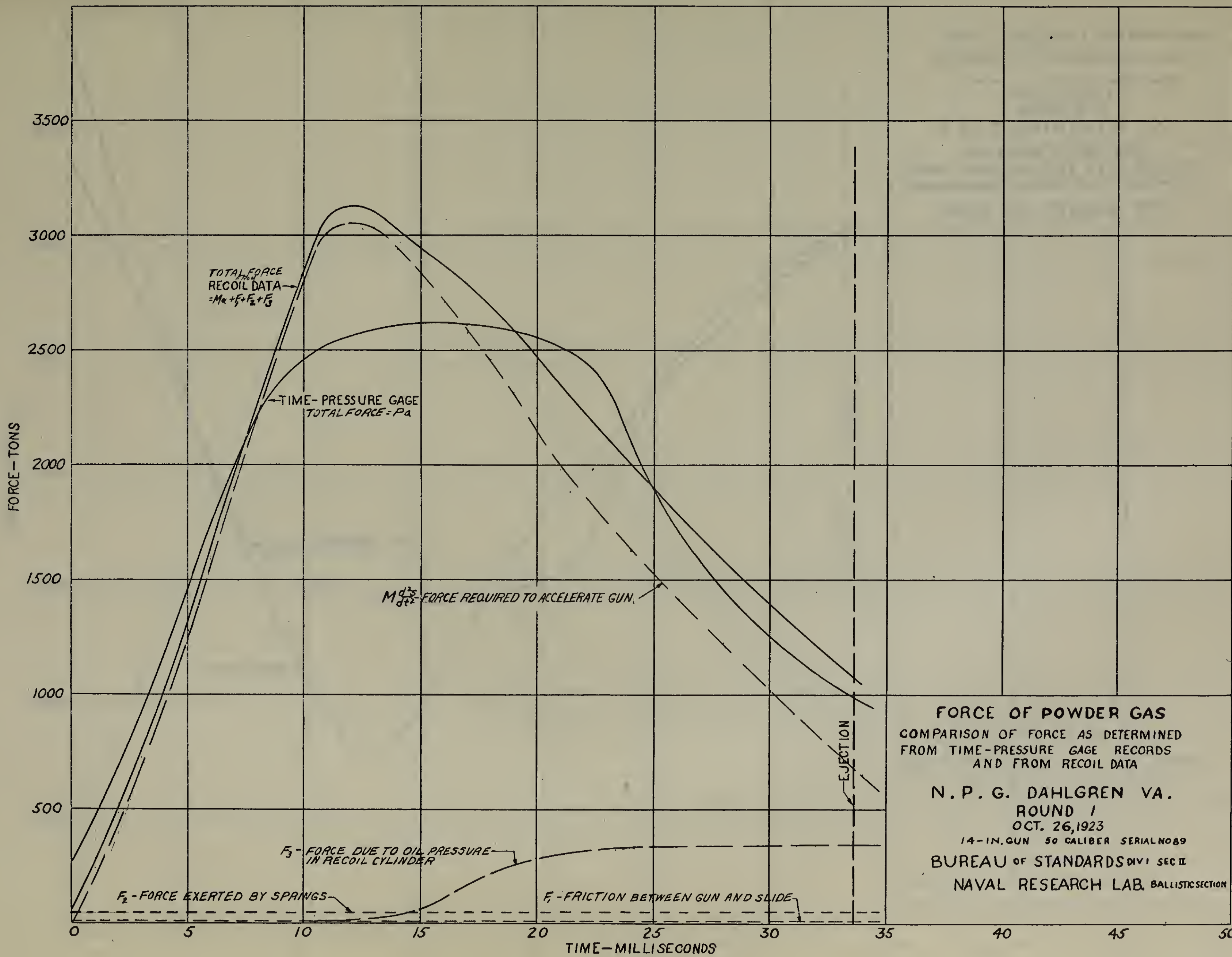

● DIRECT MEASUREMENT BY EXSPANSOMETERS AND MUZZLE FINGERS
○ INDIRECT DETERMINATION FROM MEASUREMENT OF RECOIL SPRING FORCE FRICTION BETWEEN GUN AND SLIDE AND RECOIL CYLINDER PRESSURE
PROGRESS OF PROJECTILE
THROUGH BARREL
N.P.G. DAHLGREN VA.
ROUND 6
OCT. 30, 1923
14-IN. GUN 50-CALIBER SERIAL NO. 89
BUREAU OF STANDARDS, DIV I, SEC II
NAVAL RESEARCH LAB., BALLISTIC SECTION
DISPLACEMENT - FEET
VELOCITY - FT/SEC.
ACCELERATION - FT. PER SEC. PER SEC.
VELOCITY
ACCELERATION
DISPLACEMENT
EJECTION
0
10 500 50000
20 1000
30 1500 150000
40 2000 200000
50 2500 250000
60 3000 300000
0
5
10
15
20
25
30
35
TIME - MILLISECONDS

FIG. 33



FORCE–TONS
TIME–MILLISECONDS
TOTAL FORCE FROM RECOIL DATA $= Ma + F_1 + F_2 + F_3$
TIME-PRESSURE GAGE TOTAL FORCE $= Pa$
$M\frac{d^2x}{dt^2}$ – FORCE REQUIRED TO ACCELERATE GUN.
EJECTION
F_3 – FORCE DUE TO OIL PRESSURE IN RECOIL CYLINDER
F_2 – FORCE EXERTED BY SPRINGS
F_1 – FRICTION BETWEEN GUN AND SLIDE
FORCE OF POWDER GAS
COMPARISON OF FORCE AS DETERMINED FROM TIME-PRESSURE GAGE RECORDS AND FROM RECOIL DATA
N. P. G. DAHLGREN VA.
ROUND 1
OCT. 26, 1923
14-IN. GUN 50 CALIBER SERIAL NO 89
BUREAU OF STANDARDS DIV I SEC II
NAVAL RESEARCH LAB. BALLISTIC SECTION

FIG. 34

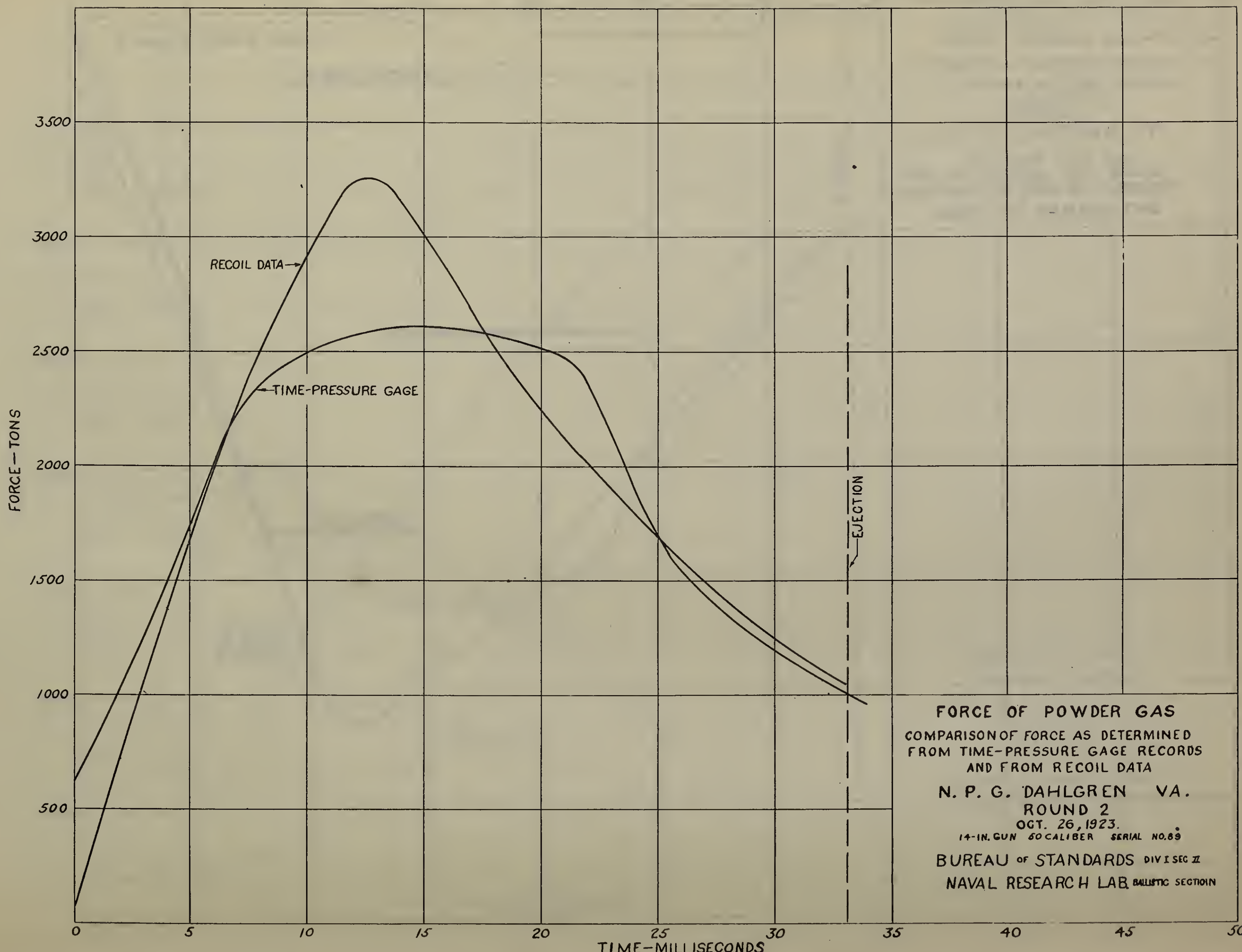

FORCE—TONS
3500
3000
2500
2000
1500
1000
500
0
5
10
15
20
25
30
35
40
45
50
TIME—MILLISECONDS
RECOIL DATA
TIME-PRESSURE GAGE
EJECTION
FORCE OF POWDER GAS
COMPARISON OF FORCE AS DETERMINED
FROM TIME-PRESSURE GAGE RECORDS
AND FROM RECOIL DATA
N. P. G. DAHLGREN VA.
ROUND 2
OCT. 26, 1923.
14-IN. GUN 50 CALIBER SERIAL NO. 89
BUREAU OF STANDARDS DIV I SEC II
NAVAL RESEARCH LAB. BALLISTIC SECTION

FIG. 35

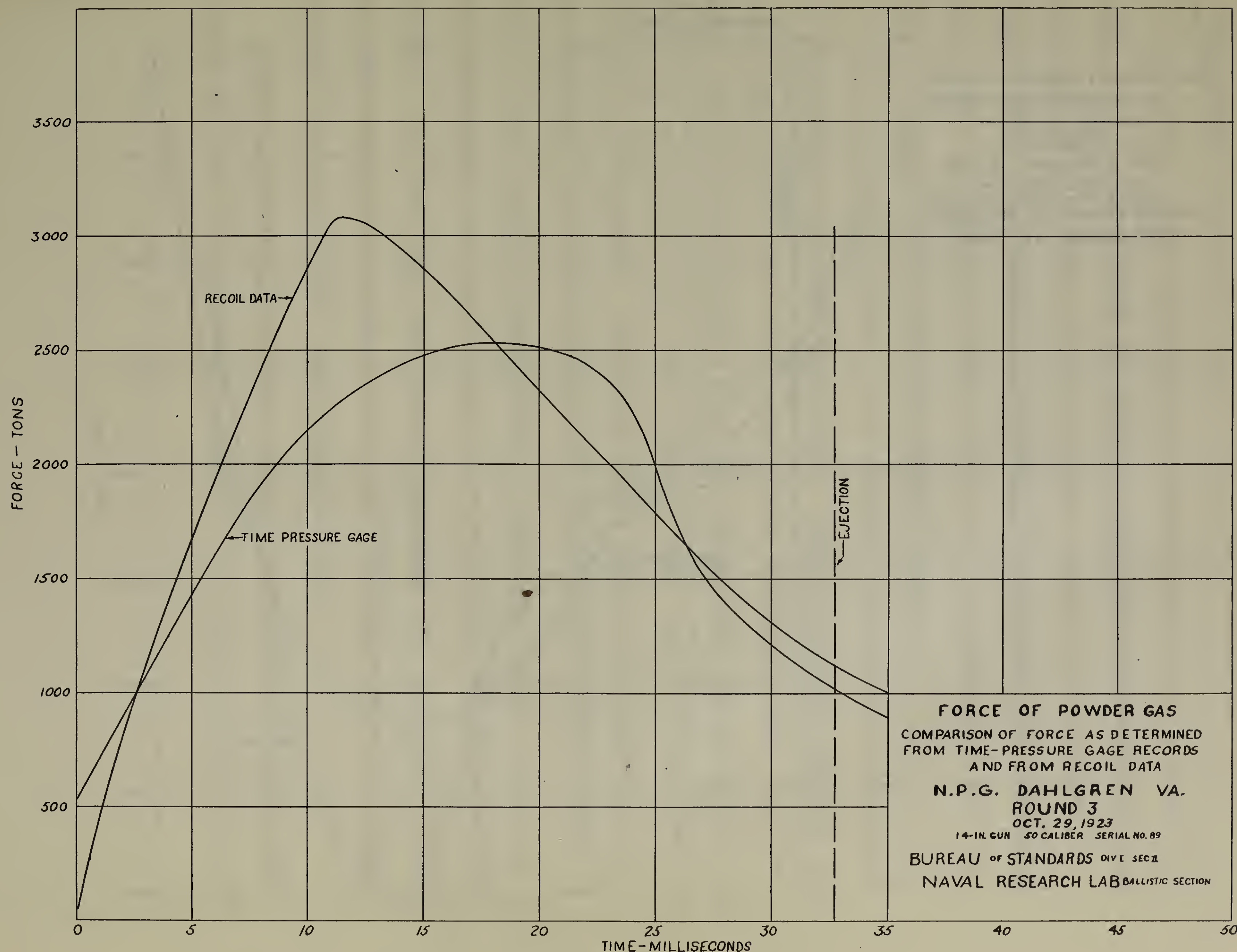

FORCE OF POWDER GAS
COMPARISON OF FORCE AS DETERMINED FROM TIME-PRESSURE GAGE RECORDS AND FROM RECOIL DATA
N.P.G. DAHLGREN VA.
ROUND 3
OCT. 29, 1923
14-IN. GUN 50 CALIBER SERIAL NO. 89
BUREAU OF STANDARDS DIV I SEC II
NAVAL RESEARCH LAB BALLISTIC SECTION
RECOIL DATA
TIME PRESSURE GAGE
EJECTION
FORCE — TONS
TIME-MILLISECONDS
3500
3000
2500
2000
1500
1000
500
0
5
10
15
20
25
30
35
40
45
50

FIG. 36

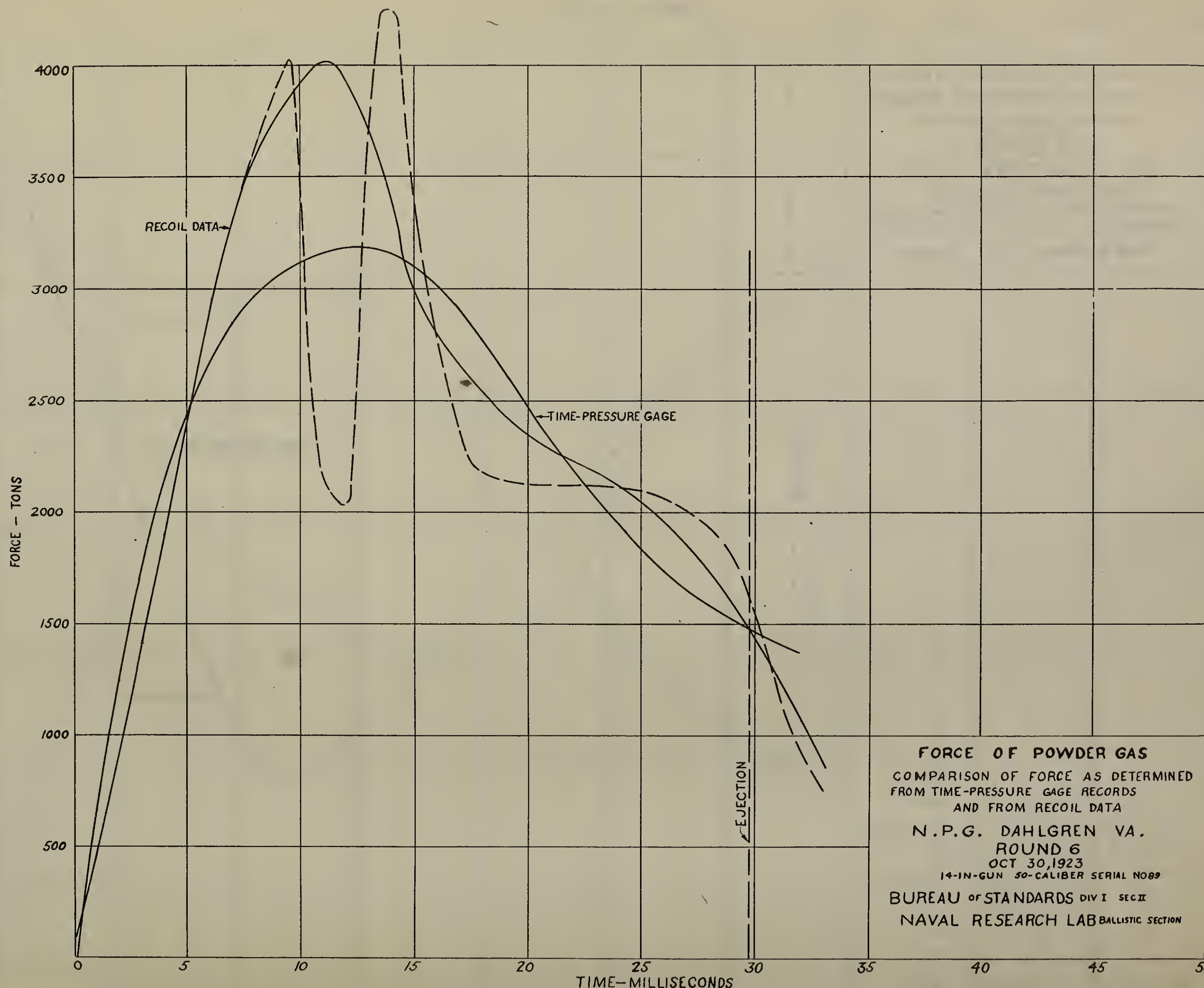

4000
3500
3000
2500
2000
1500
1000
500
FORCE – TONS
RECOIL DATA
TIME-PRESSURE GAGE
EJECTION
0
5
10
15
20
25
30
35
40
45
50
TIME–MILLISECONDS
FORCE OF POWDER GAS
COMPARISON OF FORCE AS DETERMINED FROM TIME-PRESSURE GAGE RECORDS AND FROM RECOIL DATA
N.P.G. DAHLGREN VA.
ROUND 6
OCT 30,1923
14-IN-GUN 50-CALIBER SERIAL NO89
BUREAU OF STANDARDS DIV I SEC II
NAVAL RESEARCH LAB BALLISTIC SECTION

FIG. 37

IV. Discussion of Results and Conclusions.

A. Velocity

The results of the velocity determinations by seven different methods are given in table 4 page 28. The methods are tabulated in the order of the accuracy which the different methods might reasonably be expected to possess, the correctness of the arrangement being borne out in general by the results to which the use of each method leads.

Of the methods listed, the contact screens and solenoids are undoubtedly the most reliable. Their accuracy depends only on the precision with which the screen distances and the frequency of the oscillograph fork are known and the accuracy with which the record on the oscillograph film can be translated into terms of velocity. The last-named factor is probably the one limiting the accuracy and the chief source of the small error in the present measurement, since the screen distances and fork frequency were both known to better than one-tenth of one percent. Of the two, the contact screens are probably slightly the more accurate because of the more definite type of record produced by them on the oscillograph film.

Next in order of accuracy comes the use of rotating band finger and the first contact screen. It falls slightly behind

the first two in probable accuracy because of the different type of the two events recorded on the oscillograph, and also because the measured distance must be corrected by allowing for the recoil displacement of the gun at ejection. It is, nevertheless, a method of very great accuracy.

The sources of error in velocity determination by the use of the Boulonge Chronograph have been the subject of much discussion and contention. They may be briefly summed up as follows:

1. Uncertainty in the position of the projectile when the screen wire breaks

2. Possibility of variation in the time of operation of the releasing magnets and in the time of operation of the instrument itself.

3. Inaccuracy in the determination of the time interval from the marks on the chronograph rod.

Its advantages lie in simplicity of operation and the rapidity with which the results may be determined.

On the fifth line of table 4 the average of the first four methods is given for each round in which complete data were obtained. Assuming this average to be the true velocity of the projectile and comparing the individual determinations with it, gives a fair idea of the accuracy of each. The individual differences are given in lines 12, 13, 14 and 15 of the table.

Examining these it is found that the greatest difference between the two are:

for the contact screens	3.5	f.s. or 0.12%
for the solenoids	7	f.s. or 0.25%
for the method using finger & contact screen	5	f.s. or 0.2%
for the Boulonge Chronograph	9	f.s. or 0.3%

the average difference being considerably less for any of the four methods.

Comparison of the results obtained in rounds 1, 2 and 3 all fired with service charges is interesting. It will be noted that of the four methods already discussed all except the chronograph indicate that the velocity in round 2 was slightly lower than in round 1 while round 3 was considerably lower than either 1 or 2. The chronograph velocities reported were the same for all three rounds. Turning to table 1 it will be noted that the differences in range are in the same order as that in the velocities, the range in round 1 being the greatest and round 3 ranging considerably shorter than either of the other two. Since these rounds were fired on different days and consequently under varying atmospheric conditions, the trend towards correspondence of the differences in velocity and range may not be conclusive. It would appear worth while, however, to

have the observed ranges reduced to standard meteorological conditions and then compared with the measured velocities. Neither the data nor the experience necessary to apply the corrections are available at this Bureau, but it might be done to advantage by the Proving Grounds. The differences in velocity here are very slight and the failure of the chronograph to detect them is not serious. The results do, however, exemplify the greater accuracy of the oscillograph methods.

The results of the ejection velocity determinations are shown in lines 6 to 9 of table 4. The differences between the average of the finger velocities and the "true" velocity in line 5 are given in line 16. Since the acceleration of the projectile under the action of the powder gases continues for some time after the projectile leaves the gun, the ejection velocities should not be equal to this "true" velocity but somewhat less than it. The true test of ejection velocity accuracy is not, then, the difference between the values in lines 9 and 5 but rather the consistency of these differences. Even if the total difference between the two is considered, however, the ejection velocities are found to differ from the values of line 5 by only about .8% on the average and 1.1% as a maximum. It will be noted that the ejection velocities are consistently lower than the "true" velocity. That is to say, the difference is in the right direction

Assuming then that the increase in the velocity of the projectile after it leaves the muzzle is not more than two or three percent, an assumption which is justified by the close agreement of the muzzle velocity as determined by the chronograph and the velocity measured between the rotating band finger and the first contact screen, the average ejection velocity may be considered to have an accuracy of somewhat better than 1%.

While this accuracy is not as great as has been obtained in previous tests with the same apparatus, it should be borne in mind that the conditions were peculiarly unfavorable to the use of this method. Attention has already been called to the worn condition of gun #89 used for the test as shown in figures 1 and 5. There are two distinct ways in which the wear in the gun might affect the results of the ejection velocity determinations. In the first place the possibility of the projectile emerging from the gun "cocked" or tilted, instead of straight, is greatly increased by the enlargement of the bore and the uneven wear of the lands. The effect of such tilting would be to change the distance traveled by the shell between the time it made contact with the ogive and rotating band fingers respectively, and so introduce an error in the "base line" of the measurement. Because of the curvature of the ogive, only a slight tilt of the shell is necessary to change the

"effective shell length" considerably. When as in the present experiment, more than one pair of fingers are used, the effect of this tilting can to some extent be eliminated by placing pairs of fingers nearly opposite each other so that an increase of the effective shell length at one pair will be accompanied by a corresponding decrease at the other, and the average of the two determinations will be but slightly affected by the tilt. It is probable that the differences between the individual finger determination in a given round, which, as shown in the last line of the table, amounted to as much as three percent in one instance, were due to a great extent to tilting of the shell.

The second effect of the worn condition of the gun on the accuracy of ejection velocity determinations has to do with the method used to set up the fingers. The gage used for this purpose consists of a curved bearing surface which is held inside the gun against the lands and carries concentrically-curved surfaces against which the tips of the fingers are set up when being fastened in their blocks. When such a gage is used under the conditions shown in figure 1, it is obvious that the setting of the fingers will not be as accurate as might be desired. Because of the fact that the land heights are below normal there

is danger that the rotating band fingers will be set too far out and so miss the front end of the rotating band. And because of the uneveness of the wear, the entire set-up becomes slightly eccentric with reference to the axis of the gun, producing to a less degree the same effect as that caused by a tilted projectile.

On the whole, however, the ejection velocity records may be considered as satisfactory, since they indicate that an accuracy of one percent or better may be obtained by this method, even under very unfavorable conditions, provided that a number of pairs of fingers are used.

Because of atmospheric conditions and some instrumental difficulties only two satisfactory records were obtained with the projectile camera. The velocities as determined by this method are given in line 10 of table 4, and are found to agree with the other determinations to somewhat better than one percent. The velocities were calculated from measurements made on the film by means of the formula

$$V_p = \frac{V_f}{r}\left(\frac{a}{a+d}\right)\left(\frac{l-d}{D}\right)$$

where V_f = speed of the film
r = ratio of magnification
a = distance between successive shutter openings
d = distance between two successive photographs of the shell
D = distance between two successive pictures of a stationary objection in the field.

The derivation of the formula is given in this Bureau's Report on "Work with Projectile Camera at Dahlgren, Va., dated March 1923".

The velocity of the projectile as determined from the curves of shell motion inside the gun is given on line 11 of the same table. This method is new and its features are discussed at greater length under the heading "Displacement of Projectile Inside the Gun". The results do not differ from those obtained by other methods by more than one percent, despite the fact that the method is somewhat indirect.

In considering the results of the velocity measurements, recognition would be given to the fact that the methods tried out in this experiment were of three quite distinct types and have diverse applications. The first four methods requiring as they do the use of screens situated at some distance from the gun are limited in their application to conditions similar to those existing at the proving grounds, where space and equipment for the screens are available. The ejection velocity and shell displacement methods on the other hand require no external apparatus and their development has been aimed toward providing a means of measuring velocity on shipboard. With the limitations imposed on them by that fact, chief of which is the short space interval over which the velocity must be determined it is not to be expected that they can compete with the former methods as far

as precise determinations ashore are concerned. Viewed from this standpoint the performance of the finger and shell displacement methods requires more consideration than a bald comparison of the accuracy of the methods would warrant for them. The third type of measurement, that by the camera, can also be found useful on shipboard. Its best field, however, appears to lie in the determination of velocity at some distance from the gun, as for example in plate firing or at points on the trajectory remote from the gun.

B. Pressure.

On figures 6 to 12 pages 31 to 37, are curves showing the pressure on the breech of the gun as determined by spring pressure gages. The maximum pressure as recorded by the crusher gage method is also shown on the curve sheets. Reference to the curves shows that, as in previous experiments, the maximum pressure from the time records is from 6 to 16 percent greater than that recorded by the crusher gages.

For rounds 1, 2, 3, 5, and 6, the maximum pressures were also calculated from the records of recoil displacement of the gun and of the forces opposing recoil. The pressures so determined were found to be considerably higher than those indicated by either type of direct measurement. Further discussion of the calculated pressures will be found under the heading "Force

Exerted by Powder Gas".

A comparison of the maximum pressures as determined by the three methods is given in table 5, page 29.

The results of the time pressure measurements are disappointing, and in view of the discrepancy between the results of measured and calculated pressure no great accuracy is claimed for either. In preparing for the experiment two new gages designated on the curve sheets as model II Modification A numbers 2 and 5 were made up but no opportunity was found to give them a service test beforehand. Of these two gages one, #5, developed a ground in its recording mechanism during the first round and, although its mechanical action, as indicated by the partial record obtained, appeared to be satisfactory it was found necessary to remove it from service in the second round. The second of the new gages #2 gave a satisfactory record throughout the firing but the subsequent reading of the oscillograph film showed that the piston was tending to stick because of insufficient clearance, a fault common in an untried gage and readily remedied when its existence is recognized. Because of these casualties, the curves for the service rounds were plotted practically entirely from the records of a single old gage. For use during the last four rounds another, gage Model 1 No. 3, was inserted and the later pressures are the average of the two records.

With the benefit of the experience gained in previous tests it was expected that more accurate data would be secured by the use of the new gages. Because of their failure to function, however, these expectations were not realized and it cannot be said that this work has added anything to the development of the method for pressure determination.

C. Recoil - Motions and Forces.

Curves showing the motion of the gun in recoil up to ejection are given in figures 13 to 19, pages 38 to 44. From the displacement-time curves as determined by use of the recoil-meters the usual velocity-time and acceleration-time curves have been derived by graphical differentiation.

The pressure in the recoil cylinder as determined by use of the spring gage and oscillograph is shown as a function of time in figures 20 to 25 pages 45 to 50. A comparison of the results obtained with this gage and with the engine indicator may be found in the pressure displacement curves on figures 26 and 27, and indicates that the indicator lacks sufficient sensitivity to give results of any accuracy.

The results of the other measurements made in connection with recoil, viz., the force exerted by the counter recoil springs and the friction between the gun and slide have not

been shown in curve form because neither varies appreciably during the time between the start of recoil and ejection. The magnitudes of these forces as determined statically by the method described in part II of this report are as follows:

S_0	= Force due to initial compression of springs	91500 lbs
K	= Spring constant or increase in spring force as the gun recoils	2100 lbs/in.
F_0	= Friction between gun and slide at 0° elevation	
	Initial value	25000 lbs
	Value after the gun starts to recoil	9600 lbs

These results are the average of a number of independent determinations made both before and after the experimental firing. The individual records did not vary from the mean by more than 2 or 3 percent as a maximum and the average values given are certainly not in error by as much as 1 percent.

It will be noted that two values are given for F_0 one initial and the other qualified by the statement "after the gun starts to recoil." This distinction was made necessary by the fact that in making the measurements, the angle at which the gun started to leave battery under its own weight when slowly elevated, was found to depend on the manner of its last previous return to battery. In runs made immediately after firing, in which cases the gun had come back to battery with some velocity under the action of the counter recoil springs, the angle to which it was necessary to elevate it in order to cause it to slide out

of battery was considerably greater than in subsequent runs in which the proximate return to battery had been induced by a slow depression of the gun to prevent its acquiring any appreciable velocity. In view of the tendency of the gun to wedge itself in the slide when returning to battery, the phenomemon observed might have been expected. At any rate the results show that the friction between gun and slide, is, because of the wedging action, nearly three times as great initially as it is after the gun has started to recoil.

The comparative magnitude of these forces in relation to the other forces acting during recoil is shown by the curves on figure 34. The comparison brings out the fact that they are practically negligible at the time of maximum pressure and amount to only a few percent of the total force exerted at any time up to ejection.

Returning to the time records of recoil the curves of pressure in the recoil cylinder will first be considered. For the low velocity rounds 4 and 7, the pressures were so low that the data is meagre and unimportant. The curves for the service velocity rounds and for round five which was fired at somewhat reduced velocity, present a fairly normal appearance and indicate that the pressure in the recoil

cylinders is roughly proportional to the velocity of the gun. In this they check the results of previous investigations with the same apparatus. Attention is called, however, to the fact that on the steep portion of the curves representing the time interval between one and two hundredths of a second after the start of recoil, a strict adherence to the location of the recorded points in constructing the curves would result in a kink in this portion of the curves. Under ordinary circumstances and considered by themselves, these variations from a smooth curve might reasonably be construed as representing experimental errors and, in constructing the curves for these rounds, that was the viewpoint taken.

When the data for the proof round, 6, was plotted, however, the kink was no longer a mere suggestion but an unmistakable fact and became a feature of decided significance. It is worthy of note here that the gage used in round 6 was not the same as that used in the other rounds, and that the magnitude of the pressure at which the slope of the curve fell off was different in round 5 than in the other rounds in which the same instrument was used. These facts preclude the possibility that the unexpected results might be due to any defect in the gages or to errors of calibration.

Because of the close relationship between recoil cylinder pressure and the velocity of the gun it is to be expected that whatever caused the kink in the pressure curves would also have some effect on the recoilmeter records of gun motion. Attention is therefore recalled to the curves of gun motion in figures 15 and 19. Unfortunately for the purpose at hand, the records obtained from the recoilmeter are of displacement rather than of velocity hence there is a tendency for the effect of small variations in velocity to be - as it were - integrated out of the record, and their detection from displacement curves is consequently much more difficult. For this reason it is not surprising that there should be found in the gun displacement curves no definite variations corresponding to the variations in the points on the pressure curves. For round six, however, the recoil data does show a small but definite wave in the displacement curve corresponding in time to the kink in the pressure curve and indicating that, in this case, the disturbance of the velocity was sufficiently great to affect the displacement appreciably.

Having checked back over the data completely, beginning with a rereading of the oscillograph film, in order to be sure

that the wave was not due to errors of observation, this portion of the displacement curve was replotted to a greatly enlarged scale and differentiated twice to obtain the corresponding velocity and acceleration curves. The results of this treatment of the data are shown by the dotted portion of the curves on figure 18. The solid portions represent the velocity and acceleration values obtained by ignoring the variations and considering the displacement record as a smooth curve.

The most likely, and probably the only possible, explanation of the records is that, at this point, the projectile was tending to sick in the gun. "Sticking" is perhaps too strong a word because the curves on figure 18 do not show that the shell at any time lost velocity or even lost all its acceleration. The cause of the variations might, however, be better described by the statement that at this point, the friction between the projectile and the gun was, for an instant, enormously increased. Sticking of the projectile, even to the extent that the lining of the gun was damaged, is not an unknown occurrence and the condition of the gun used in the present experiment, as shown in figure 5, was particularly favorable for it. Reference to figure 5 shows that the bore of the gun was considerably enlarged near the breech and muzzle but nearly normal in diameter at the middle. The approximate position of

the projectile in the gun at the time when the friction between the two was at maximum is denoted by the vertical line S S'. It will be observed that the maximum friction and consequent loss of acceleration occurred, not where the bore was smallest, but where it was converging most rapidly.

Some idea of the magnitude of the friction between shell and gun necessary to produce the conditions represented in figure 18 may be gained from the force-time curve for this round in fig. 37. Assuming that the force curve calculated from the recoil data discussed above is sufficiently accurate to give a fair representation of the existing conditions, the difference between the normal friction between gun and shell and the abnormal friction causing the disturbance is equal to the difference between the ordinates of that curve at its maximum and minimum points. The curve referred to is again the dotted one and the value of the momentary increase in friction so determined is approximately 2500 tons. This figure should of course be considered as nothing more than an estimate because of the indirect method by which it was determined.

In view of the unusual features of the data discussed in the preceding paragraphs, and the uncertainty introduced into our attempted analysis of them by reason of the lack of a direct

determination of gun acceleration the necessity for the development of an instrument suitable for such direct acceleration measurement is brought home with renewed emphasis. The desirability and advantages of such an instrument have been long recognized but to date no real progress has been made in its development. Such an instrument would be of the greatest value in studying the problems of interior ballistics since it would afford at the same time a record of the motion of the gun and a check on the internal pressure. It is therefore to be hoped that this method of measurement will be no longer neglected but that plans for its development be given careful consideration in laying out the program for future ballistic work.

D. Displacement of Projectile in Gun.

When the records obtained in the experiment were assembled for analysis after the firing the value of the expansometer records was entirely a matter of conjecture. Some simple method of investigating their accuracy consequently seemed desirable, and since, as a first approximation, the displacement of the projectile may be considered to be proportional to that of the gun, in the inverse ratio of their masses, the data were first compared with the recoilmeter records of gun displacement on that assumption. To make this comparison the chart on figure 28 was constructed. A series of straight lines on the

chart was drawn between the two points in each round, at which the displacement of the shell relative to the gun were known from other sources; viz., at the start of recoil where both are zero, and at ejection where the relative displacements of the two are known from muzzle finger and recoilmeter measurements. The intermediate points were then determined by combining the expansometer and recoilmeter records and plotted on the chart.

Each point on the chart is marked with its round number and the points are found with a very few exceptions to fall close to the straight lines. Inspection of the chart then showed that the expansometer records were to this first approximation at any rate satisfactory. The chart, however, showed more than that, because of the fact that the expansometer points almost invariably fell below the corresponding straight line, particularly in the middle of the chart. It will be recalled that the assumption in which the straight line relationship was based is only an approximation, and that the lines on the chart accurately represent the relative displacement of the gun and shell at their two ends. And since the relationship between the two displacements would be more

precisely represented by curves slightly concave instead of straight, the expansometers were concluded to be more accurate than the rough method used to check them.

The reliability of the expansometer records being thus established, a more searching analysis of the results obtained by their use was then undertaken. From the fundamental equations of internal ballistics the relation between the displacement of the projectile and that of the gun at any instant may be expressed by the following formula which while not exact has been found sufficiently accurate for most purposes.

$$(1)\quad (M+m+m')\,Y + \iint F_1\,dt\,dt + \iint F_2\,dt\,dt + \iint F_3\,dt\,dt = \left(m + \frac{m'}{2}\right) y.$$

in which

M = Mass of the gun
m = Mass of the projectile
m' = Mass of powder or of the mixture of powder and gas
Y = displacement of the gun in recoil
F_1 = force of friction between the gun and slide
F_2 = force exerted by the counter recoil springs
F_3 = force of the recoil cylinder
and y = the displacement of the projectile along the bore of the gun

The next step in the treatment of the expansometer records was to examine them in connection with this equation. All of the terms in the equation can be evaluated from the data secured in the experimental firing, since Y is known from the recoilmeter records, F_1 and F_2 by static measurement and F_3 from the records of recoil cylinder pressure. It was therefore possible to construct and compare curves representing each side of equation (1) as a function of time, the values for the left-hand side being known for any time between the start of recoil

and ejection, and those for the right-hand side at ten points where the value of y was recorded, eight by the expansometers and two by muzzle fingers.

These curves were constructed then for rounds 1, 2, 3, 5 and 6 and while the two curves for a given round differed by only a few percent at any point, the values of y as determined from the expansometer records were slightly but consistently higher than those arrived at by using the recoil data.

This fact indicated the need for a more precise equation to represent the relation between the displacements. Starting with the assumption that the density of the powder and gas combined may be considered uniform at any instant and taking into account the effect of the size and shape of the powder chamber the following formula was derived.

$$(2) \quad (M+m+m')\ Y + \iint F_1 dtdt + \iint F_2 dtdt + \iint F_3 dtdt = (m+\frac{m'}{2})(1-\frac{A}{k+y})\ y$$

which may also be written in the form

$$(2a) \quad y = \frac{(M+m+m')\ Y + \iint F_1 dtdt + \iint F_2 dtdt + \iint F_3 dtdt}{m + \frac{m'}{2} \quad (1 - \frac{A}{k+y})}$$

In which A and k are constants depending only on the dimensions of the gun and the remaining terms are the same as in equation (1).

The complete derivation of this formula and a discussion of its accuracy are given in the Appendix to this Report.

Using this equation the curves in figures 29-33 were constructed. The solid circles represent values of y obtained by direct measurement using the expansometers and muzzle fingers,

the open circles the values calculated from recoil data by use of formula (2a). The agreement between these two independent determinations of projectile displacement is remarkable. Neglecting the records of expansometer #8, the one nearest the muzzle, which is found to give results consistently at variance with the others and with the muzzle fingers, a more representative curve could scarcely have been drawn through the expansometer points than the one formed by the points representing the results of the indirect measurements.

With reference to the operation of the expansometers it was found that for rounds fired at reduced velocity, the three expansometers farthest back from the muzzle failed to function. Beginning with expansometer #4 which was located about 32 feet from the origin of the rifling, practically complete records were obtained throughout the experimental firing. The failure of the instruments more remote from the muzzle at low pressure was doubtless due to the greater thickness of the gun toward the breach and the fact that when the pressures in the gun were below normal the expansion of such points was insufficient to operate the instruments. For this experiment the expansometers were initially adjusted to operate on service rounds and no change was made in the initial adjustment during the test. It is possible that, by changing the adjustment when data under conditions of low internal pressure are desired, the instruments set farther back on the gun may be made to function even at the lower pressure.

Reference has already been made to the peculiar characteristics of the records of expansometer #8. This instrument was located less than three feet back from the muzzle and apparently opened too early in every round fired. Whether this behavior was due to faulty action of the instrument itself or to abnormal conditions existing near the muzzle, either because of gas leakage past the projectile or the reflection and amplification of some vibration from the end of the gun is problematical. This point may be settled by interchanging the instruments in future experiments.

The worst performance made by any of the other expansometers was that of expansometer #1 which gave a record falling almost 6 percent above the mean curve in round 1 and approximately 4 percent below it in round 2. The records of the intermediate expansometers show a variation of the individual records from the curves of less than one percent on the average, and not over twice that as a maximum.

Because of the comparatively few records obtained at low pressures, displacement-time curves have not been constructed for rounds 4 and 7. The pressures represented by the curves which were constructed vary from less than 20,000 lbs/sq.in. in round 5 to over 40,000 in round 6, with the corresponding velocities 2150 to approximately 3000 feet/second. It may be concluded therefore that the operation of the expansometers is satisfactory over a fairly wide range of pressure and velocity.

From the displacement curves, by successive graphical differentiations, the velocity and acceleration of the projectile

inside the gun have been determined. These curves are also shown in figures 29-33. Since the expansometer records do not begun until some 15 or 20 thousandths of a second have elapsed after the start of recoil, these curves are representative rather of recoil data than of the expansometer results, particularly during the first half of the record. The similarity of the curves of gun motion and shell motion for round 6 should not therefore, be construed as showing anything new in connection with the friction between projectile and gun discussed when considering recoil.

The difference in appearance between the acceleration curves of the shell and gun in a given round are interesting but, because of the inaccuracies inherent in the indirect method of determining acceleration, the significance of these differences is open to question. The velocity of the projectile at ejection as determined from the curves of shell motion after correction for the velocity of the gun, is shown in table 4 and found to agree fairly closely with the velocity determined by other methods. Because of the failure of the expansometer nearest the muzzle to function satisfactorily, no attempt has been made to calculate the velocity near ejection directly from the expansometer records.

E. Forces

The forces acting on the gun and projectile at any instant are defined by the following well-known equations for the gun

$$(3) \quad PA - P'A' = M\frac{d^2Y}{dt^2} + F_1 + F_2 + F_3 + F'$$

and for the projectile

$$p\,a = m\frac{d^2 y}{dt^2} + F'$$

where

A = cross sectional area of the breech exposed to the powder pressure

A' = effective cross section of the forward end of the powder chamber on which the pressure acts in the direction opposite to the recoil of the gun

a = the area of the bore of the gun or the base of the projectile

and where at any time t

P = powder pressure at the breech of the gun

P' = powder pressure at the forward end of the powder chamber

p = powder pressure at the base of the projectile, all the remaining terms being the same as used in equations (1) and (2).

A comprehensive investigation of the problems of internal ballistics, then, involves the determination either by calculation or actual measurement of all the terms in these equations. Certain of the factors involved are, however, by nature extremely difficult to measure. No apparatus has yet been devised which will directly indicate even approximately the friction between the gun and the projectile, and the determination of P' and p, also, present difficulties which are still far from being solved.

The practical aim of any ballistic program must be, therefore, the selection of the factors which are the most reliable for purposes of analysis leading to the indirect determination of the remaining factors. It is with this purpose in view, that the work done by this Bureau has been directed along the lines in which the development work has been carried out.

In the present experiment, determinations of varying accuracy were made of all the factors involved in equations (3) and (4) with the exception of the three referred to above. In order to make these data applicable to a complete solution of the equations the hypothesis of constant density of the propelling charge was developed mathematically as shown in the Appendix, to obtain an equation in which the pressures F' and p are expressed in terms of the other factors and of the dimensions of the gun. This done, the only unmeasured variable remaining is F' which may then be evaluated by use of the equation. The solution is given in the following formula.

$$(5)\quad pa = \frac{\left\{m+m'\left(1+\frac{C_4}{(k+y)^2}\right)\right\}}{2}\frac{d^2y}{dt^2} - \left\{m+m'\left(1-\frac{k-l-h}{k+y}\right)\right\}\frac{d^2Y}{dt^2}$$

$$-\frac{m'C_4}{2(k+y)^3}\left(\frac{dy}{dt}\right)^2 - F'$$

where C_4 , k, l, and h are constants depending solely on the dimensions of the gun.

The agreement of the measured displacement of the projectile with the values calculated from the displacement formula (formula 3) is a fair indication of the validity of formula (5), since both were derived from the same single fundamental assumption. If then sufficiently accurate records were available of the pressure at the breech of the gun and of the acceleration of the gun and projectile, the value of F' could be determined and the solution would be complete.

Unfortunately the failure to secure more reliable pressure

records makesit impossible to carry through the determination for the friction between gun and shell at this time, since the pressure term is the most important one in equation (5). The successful use of the formula also requires accurate records of gun acceleration and it is doubtful whether the present method of determining this factor from recoilmeter measurements would prove adequate in any event.

Consequently it has been found necessary to fall back on equation (3) making use of the approximation that PA - P'A' = Pa. This is the same as considering the pressure to be the same at both ends of the powder chamber and neglecting the conical shape of its forward end, the error introduced by which assumptions being similar than the discrepancies apparent in the pressure records.

Force curves constructed from this approximation of formula (3) are shown in figures 33-37. From these curves the relative magnitude of the external forces and the internal pressure can be obtained. They show that the friction between the gun and slide is negligible and that none of the external forces are appreciable until after the time of maximum pressure.

The truncated appearance of the Pa curves is due to the sticking of the gage pistons referred to in section B of this part of the report. The pressures as determined from the force curves constructed from recoil data are given in table 5. Since this method of determining pressure neglects entirely the friction between shell and gun the values obtained by it should be too low. They are nevertheless considerably in excess of the

values recorded by the time pressure gages. They are probably the best values obtained in the experiment, however.

It would seem unwise to draw any conclusions as to the value of the friction between the gun and shell from the curves on figures 33-37. Toward the end of the record where the curves have the most accuracy the indications are that this friction is small since the curves lie close together. A strict interpretation of the curves would indicate a negative friction at some places. This absurdity is of course due to the inaccuracy of the data. An estimate of the increase in the friction under the abnormal conditions in round 6 has already been given in discussing the recoil records.

From the considerations set forth in the preceding paragraphs, it must be concluded that the investigation of internal ballistics is faced by the need for two instruments of vital importance:

(a) a satisfactory time pressure gage

and (b) an instrument suited to the direct measurement of gun acceleration.

Of the two the pressure gage is already well along in its development and the difficulties remaining to be overcome are mostly of a mechanical nature. Little or no progress has been made with the latter instrument, however, and its development is a matter which should be given serious attention.

V. Summary of Results and Conclusions.

Despite admitted deficiencies in some details of the data it is believed that the firing reported herein constitutes the most comprehensive and fruitful single experiment yet undertaken in the study of the general problems of internal ballistics. In it data of unusual significance were obtained in a number of instances and the opportunity offered for correlating the records and checking the results of diverse methods of measurements has led to conclusions of great value and interest. The chief features of the data may be briefly summarized as follows:

1. Velocity - Of the methods of velocity measurement adapted to precise work ashore, the results obtained by the use of the contact screens and solenoids are found to possess a degree of accuracy sufficient for all practical purposes. There is little to choose between these two methods, the contact screens giving if anything a slightly greater accuracy.

The ejection velocity determinations, while naturally falling somewhat below those mentioned above in accuracy indicate that this method has been brought to a stage of development where reliable results may be obtained even under adverse conditions.

2. Pressure. - No advancement in the development of pressure recording is shown by this experiment. The results are valuable chiefly in emphasizing the necessity for improvement in the mechanical features of the gages.

3. Recoil

a. External Forces Acting During Recoil - A complete time record of the external forces incidental to recoil of the gun was for the first time obtained. A simple method developed for the determination of spring force and friction between the gun and slide, makes it unnecessary for the future to rely on approximate data on these forces.

The records of pressure in the recoil cylinder were found to show not only the magnitude of the force acting, but also, to be sufficiently precise to detect the existence of abnormal conditions existing within the gun, this offering a check on the records of gun motion.

b. Motion of the Gun - The recoilmeter measurements made in this experiment are chiefly interesting in their relation to the records of projectile displacement and recoil forces. The possibilities of the combined measurements are well exemplified by the results obtained in the proof round in which some abnormal condition inside the gun are indicated by the records both of recoil and recoil cylinder pressure. That the unusual records were caused by a brief but comparatively enormous increase in the friction between the projectile and the gun, i.e., by a tendency of the projectile to stick at a point in the bore, there can be but little doubt.

The analysis of this portion of the data shows the desirability of proceeding with the development of an instrument suitable to the direct measurement of gun acceleration.

4. Displacement of Projectile in Gun.

By use of the expansometers a direct measurement of the

displacement of the projectile inside the gun was secured for the first time, so far as is known, by a method in which the apparatus was entirely external. The close agreement of the expansometer records, and the values calculated from recoil measurements is gratifying evidence of the accuracy of the one and the soundness of the other. The advantages of the expansometer method may be enumerated as follows:

a. Simplicity of the Apparatus.
   The apparatus is by no means complex. It can be installed on any gun with little effort. The gun need not be pierced nor otherwise damaged.

b. Simplicity of Operation
   The expansometers once installed are practically automatic in their operation. All tests may be conducted from the laboratory or some other remote point. During the experiment not a single adjustment was necessary in any of the eight expansometers used.

c. Simplicity of Interpreting Results
   The record given by the expansometers shows directly the displacement of the shell as a function of time. No intermediate calculation is necessary.

5. Forces

The chief value of the section of the report dealing with forces lies in the exposition of the present status of ballistic measurements and in pointing out the lines along which future work should be conducted in order to obtain a complete solution.

6. Theory

The new formulas for internal ballistics used in this report, the development of which is given in the Appendix make possible a better correlation of experimental records. The usefulness of these formula is apparent from their application to the present data.

7. Future Development

The results obtained in this experiment indicate clearly the lines along which a ballistic program should be directed. Two vital factors require careful attention.

(a) The completion of development work on the powder pressure gage.

(b) The development of an instrument for the direct measurement of gun acceleration.

The pressure gage work is already being prosecuted vigorously by the Naval Research Laboratory and the Proving Grounds, and the improvement in mechanical operation of the gage, which appears to offer the sole remaining difficulty, should be speedily obtained.

The acceleration measuring device, however, has not to date been given the attention it deserves and its development is a matter which should no longer be neglected.

Once these two instruments have been brought to a really satisfactory stage of development, the friction between the gun and shell can be determined with an accuracy far surpassing anything so far attainable. The importance of this friction cannot be denied in view of the results already obtained.

The final step would then consist of a repetition of this experiment, which with the advantages of improved gages, should prove to be by far the most valuable work ever attempted in the field of internal ballistics.

27 Appendix

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Appendix B 27

Ballistic Formulas and their Derviation

October, 1924

Numerical Test of the Ballistic Equations of the Dahlgren Firing of 1923

C. Snow (1942)

I. Introduction

The data here discussed are obtained from field and laboratory notes of the experimental firing of a 14"-50 calibre naval gun Mark IV, Mod. 3, Serial No. 89L2. The general results were given by the Bureau of Standards to the Naval Bureau of Ordnance in a report entitled "A Correlation of Diverse Ballistic Records" in Oct. 1923. We consider only the seven experimental rounds with varied charges for which fairly complete time records were obtained of the recoil $Y(t)$ of the gun, the powder pressure, and the external forces. From these by the principles of dynamics with a reasonable assumption as to the mean velocity of the burning powder and gases, the relative displacement $y(t)$ and velocity $\dot{y}(t)$ of the projectile may be found as well as the frictional force $F(t)$ between the gun and the shell. Estimates are also obtained of the force F_s necessary to start the shell and of the time interval that elapses between its start and the beginning of recoil.

The travel of the shell and its velocity thus computed is compared with observations at time t_e of ejection of the shell, since the former must be equal to length of the rifling and the relative velocity at ejection was measured by means of three pairs of contact fingers at the muzzle, one of each pair making contact with the shell near the ogive, the other at the front end of the rotating band. In addition the travel of the shell was observed by a series of expansometer hoops around the barrel whose electrical circuits were opened by the bulge that travels along near the base of the shell.

The internal energy of the burning powder, its rate of burning as it depends upon pressure and size and shape of powder grains, etc. are subjects properly belonging to internal ballistics, but are quite outside this discussion. The dynamical behaviour of the burning powder in the large, and on the average, is represented for the present argument by two things. The first is the reading $P(o,t)$ of the pressure gage in the breech and the second is the inertial effect of the fluid (powder and gas together) as it is influenced by the initial and the changing shape of its volume.

This inertia cannot be altered by any chemical transformation. Hence the following discussion of the fluid motion has for its aim to find, from the conservation of mass and momentum, the space variations of powder pressure which must accompany accelerations of the fluid and the shell. By relating the instanteous position of the center of gravity of the powder to the travel of the shell an expression is then obtained for the pressure on the shell which shows it is less than the recorded pressure at the breech by a small amount depending upon the acceleration and travel of the shell. From this the friction F(t) between gun and shell may be found, but the degree of confidence we may have in this evaluation of the unobservable friction will depend upon the closeness with which the system of equations predicts observable magnitudes such as travel and velocity of shell.

II. The Three Equations of Motion

The approximate shape of the powder chamber is shown (not to scale) in fig. 1. The length ℓ = 74.34" = 6.195 ft. of the cylindrical part has a radius b = 8.25" = .687 ft. Then it tapers with a conical region to the radius c = 7" = .583 ft. of the bore. The axial length of this conical section is h = 20" = 1.667 ft.; the length of the rifling is 49.8 feet.

Let $\mathcal{L}(t)$ denote, at time t, the total axial component, reckoned positive in the direction of motion of the shell, of the force on the gun due to powder pressure on the surface of the powder chamber. If P(z,t) in general denotes the mean pressure taken at time t over any plane circular section of the powder, which is distant z from the plane of the breech, then the pressure on the base of the shell where z = y(t) is P(y,t), y(t) being the distance of the base of the shell from the breech at time t, so that $\dot{y}(t) = \frac{d}{dt} y(t)$ = relative velocity of shell to the gun. The mean pressure over the breech is P(o,t) and this is considered a known function of the time, found from pressure gages in the breech.

The total external force D(t) is taken as positive in the direction opposing recoil, and is known with a certain precision over the interval from start of recoil t = o to ejection time t_e = .03 seconds. The velocity and acceleration of the shell relative to axes fixed in the earth are $\dot{y}(t) - \dot{Y}(t)$ and $\ddot{y}(t) - \ddot{Y}(t)$ respectively. If $\bar{y}(t)$ denotes the distance at time t from the breech to the center of gravity of the fluid (powder and gas together) then its velocity and acceleration are respectively $\dot{\bar{y}}(t) - \dot{\bar{Y}}(t)$ and $\ddot{\bar{y}}(t) - \ddot{\bar{Y}}(t)$.

Hence the three equations of dynamics are

$$1)\quad \frac{M}{g}\ddot{Y}(t) = \pi b^2 P(o,t) - \mathcal{R}(t) - F(t) - D(t)$$

$$2)\quad \frac{m}{g}\left(\ddot{y}(t) - \ddot{Y}(t)\right) = \pi c^2 P(y,t) - F(t)$$

$$3)\quad \frac{\bar{m}}{g}\left(\ddot{\bar{y}}(t) - \ddot{Y}(t)\right) = \pi b^2 P(o,t) - \mathcal{R}(t) - \pi c^2 P(y,t)$$

Subtracting equation (1) from (2) gives

$$4)\quad \frac{m}{g}\ddot{y}(t) = \frac{(M + m)}{g}\ddot{Y} + D(t) - \left[\pi b^2 P(o,t) - \mathcal{R}(t) - \pi c^2 P(y,t)\right]$$

The pressure terms here represent the total force accelerating the fluid, so that by (3) this equation may be written

$$5)\quad \frac{m}{g}\ddot{y}(t) + \frac{\bar{m}}{g}\ddot{\bar{y}}(t) = \frac{M'}{g}\ddot{Y}(t) + D(t) \text{ -------} (M' = M+m+\bar{m})$$

where the time is measured in seconds and
M = the effective mass of the unloaded gun in pounds.
m = the mass of the shell = 1400 lbs
$\bar{m}$ = the mass of the powder charge = 484 lbs for service rounds, but this was varied from 230 to 518 lbs in the experimental firing.

6a) $M' = M + m + \bar{m}$ = the effective mass of the loaded gun. If all displacements are in feet and the forces in pounds weight, then $g = 32.2$. Since pressures are in pounds per sq inch the radii b and c in these equations must be in inches. The effective mass of the gun is

$$6b)\quad M = M_0 + \frac{m_s}{2}$$

where M_0 is the mass of the unloaded gun and m_s is the mass of the recoil springs which is very small compared to M_0. Under these circumstances since one end of the spring is fixed to earth, the other end to the gun and moving with it, the velocity and acceleration at any point of the spring is proportional to its distance from the fixed end. Hence the average acceleration of the spring is half that of the gun and therefore half the mass of the spring is added to the true mass M_0 of the gun to give its effective mass (when unloaded). This argument is based upon the fact that the

mass of the springs is only 4 percent of that of the gun and the velocity and acceleration are small compared to those belonging to free vibrations of the spring. If this were not the case and the masses were comparable, the correction would be more nearly $\frac{m_s}{3}$.

All entries in the original field notes of 1923 place the mass of the springs as
m_s = 7116 lbs
In two places the total mass of those parts of the hydraulic brake (recoil pistons, plungers, etc.) which move with the gun are placed at 2242 lbs. The masses in two places are itemized as follows:

6c)

Gun = 191,860 lbs — (Apparently this includes the 2242 lbs)
Yoke = 12,145
Breech Block = 1,766
M_o = 205,770 lbs (In calculating friction M_o was taken as 206,000 lbs)
$\frac{m_s}{2}$ = 3,558

$M = M_o + \frac{m_s}{2} =$ 209,330 = effective mass of unloaded gun. (In the report M was taken as 208,000 as with a spring correction $\frac{m_s}{3}$).

The mass of the shell was m = 1,400 lbs for all rounds. The powder charge for service rounds was 484 lbs. Hence the effective mass M' = M+m+m' for the loaded gun is

6d)
M' = 211,200 lbs for Round 1,2,3($\bar{m}$=484 lbs) and for Round 6, $\bar{m}$ = 518
= 211,100 lbs for Round 5 ($\bar{m}$ = 368)
= 211,000 lbs for Rounds 4 and 7 ($\bar{m}$ = 230 and 240)

Examination of the recoil data shows the remarkable fact that during more than half of the time from start of recoil to ejection the recoil may be represented very accurately by the equation $Y(t) = At^{\alpha}$. By plotting log Y against log t, the greater part of the curve is a straight line but the deviations are important before ejection. It is then found that throughout the entire recoil period to ejection, the recoil is given with error almost imperceptible (less than .02" at any point) by the empirical equation

7a) $Y(t) = \frac{At^{\alpha}}{1 + Bt^{\alpha}}$ where $2 < \alpha < 3$

so that

7b) $\dot{Y}(t) = \frac{\alpha A t^{\alpha - 1}}{(1 + Bt^{\alpha})^2}$

and

7c) $\ddot{Y}(t) = \dfrac{\alpha A t^{\alpha-2}\left[\alpha - 2 - (\alpha+2)\, B t^{\alpha}\right]}{(1 + Bt^{\alpha})^3}$

The derivatives $\dot{Y}$ and $\ddot{Y}$ may be computed by $(7)_b$ and $(7)_c$ as accurately as Y by $(7)_a$ and with very little labor. The formulas $(7)_b$ and $(7)_c$ represent the derivatives of the function Y defined by $(7)_a$ with all desirable accuracy. But if $(7)_a$ differs from observed recoil everywhere by less than 1/2 of 1 percent it does not follow that $(7)_b$ and 7_c will represent recoil velocity and acceleration with any such precision.

III. The Velocity and Density of Powder and Gas

If c is the radius of the bore and r(z), for $o < z < \ell + h$, is the radius of the powder chamber where r is any continuous function of z such that $r(o) = b$ and $r(\ell + h) = c$ the volume V of the powder chamber is

8a) $V = \pi \int_0^{\ell+h} r^2(z)\,dz$ The total charge $\bar{m}$ is confined to this volume before the shell starts, for $t \leq -\tau$.

8b) $\bar{m} = \int_0^{\ell+h} \rho(z, -\tau)\,dz$

where in general $\rho(z,t)$ is the linear density of the fluid (powder and gas together in a statistical sense).

The mass m(z,t) which at any instant t is contained between the plane of the breech z = 0, and a plane distant z from the breech (whether this plane be fixed or moving) is

9a) $m(z,t) = \int_0^{z} \rho(z_1, t)\,dz_1$ (which gives $m(y,t) = \bar{m}$)

The distance $\eta(z,t)$ from the breech to its center of gravity is defined by

9b) $m(z,t)\,\eta(z,t) = \int_0^{z} z_1\, \rho(z_1, t)\,dz_1$ (when z = y this gives $\eta(y,t) = \bar{y}(t)$.

The volume density ρ_v may be considered a function of z and t since variations of density at all points in the same plane may be considered negligible. Then

9c) $\rho(z,t) = \pi r^2(z)\,\rho_v(z,t)$

To express the fact that mass is neither created nor destroyed at any point at any instant let $u(z,t)$ denote the mean fluid velocity relative to the gun. If the plane z then moves say $z = z(t)$ with the mean relative fluid velocity $\dot{z}(t) = u(t)$ it will remain the boundary of a fixed mass of fluid so by (9a)

$$m\big(z(t),t\big) \equiv \int_0^{z(t)} \rho(z,t)\,dz_1 = \text{constant with time, or}$$

$$\frac{d}{dt}m\big(z(t),t\big) = 0 = \rho(z,t)\,u(z,t) + \int_0^{z} \dot{\rho}(z,t)\,dz, \text{ or}$$

$$10a)\quad u(z,t) = -\frac{1}{\rho(z,t)}\int_0^{z} \dot{\rho}(z,t)\,dz$$

which is an integral form of the equation of continuity. Its differential form is the partial differential equation which results from applying $\frac{\partial}{\partial z}$ to ρ times this

$$10b)\quad \dot{\rho}(z,t) + \frac{\partial}{\partial z}\big[\rho(z,t)u(z,t)\big] = 0$$

The mean relative fluid velocity $u(zt)$ vanishes at the breech where $z = 0$ and is equal to the shell's velocity $\dot{y}(t)$. Assuming that it varies linearly gives

$$11)\quad u(z,t) = \dot{y}(t)\frac{z}{y(t)} \text{ ----------- } 0 < z < y(t)$$

With this u the equation of continuity (10b) becomes

$$12)\quad \frac{\partial \rho}{\partial t} + \frac{\dot{y}}{y}\frac{\partial}{\partial z}(z\rho) = 0$$

Let $\phi(x)$ be a function of its variable x defined only for the range $0 < x < 1$ which is arbitrary except that $\phi'(x)$ and $\phi(x)$ are continuous and

$$13)\quad \phi(0) = 0, \quad \phi(1) = 1$$

Then the general solution of the partial differential equation (12) may be taken

$$14a)\quad \rho(z,t) = \bar{m}\frac{\partial}{\partial z}\phi\left(\frac{z}{y(t)}\right) = \frac{\bar{m}}{y}\phi'\left(\frac{z}{y}\right)$$ so that (9a) and (9b) become

$$14b)\quad m(z,t) = \bar{m}\,\phi\left(\frac{z}{y}\right)$$

$$14c)\quad m(z,t)\eta(z,t) = \bar{m}\,y(t)\int_0^{\frac{z}{y}} x\phi'(x)\,dx$$ which gives

To express the fact that mass is neither created nor destroyed at any point at any instant let $u(z,t)$ denote the mean fluid velocity relative to the gun. If the plane z then moves say $z = z(t)$ with the mean relative fluid velocity $\dot{z}(t) = u(t)$ it will remain the boundary of a fixed mass of fluid so by (9a)

$$m\left(z(t),t\right) \equiv \int_0^{z(t)} \rho(z,t)dz_1 = \text{constant with time, or}$$

$$\frac{d}{dt} m\left(z(t),t\right) = 0 = \rho(z,t)\, u(z,t) + \int_0^{z} \dot{\rho}(z,t)dz, \text{ or}$$

10a) $$u(z,t) = -\frac{1}{\rho(z,t)} \int_0^{z} \dot{\rho}(z,t)dz$$

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14a) $$\rho(z,t) = \bar{m}\frac{\partial}{\partial z}\phi\left(\frac{z}{y(t)}\right) = \frac{\bar{m}}{y}\phi'\left(\frac{z}{y}\right)$$ so that (9a) and (9b)

become

14b) $$m(z,t) = \bar{m}\,\phi\left(\frac{z}{y}\right)$$

14c) $$m(z,t)\eta(z,t) = \bar{m}\, y(t)\int_0^{\frac{z}{y}} x\phi'(x)dx$$ which gives

when $z = y$

14d) $\bar{y}(t) = \frac{\bar{y}(-\tau)}{\ell + h}\, y(t)$ where $\bar{y}(-\tau) = (\ell+h)\int_0^1 x\phi'(x)dx$

The assumed fluid velocity u is, by (11), everywhere zero when $t = -\tau$ ($\dot{y}(-\tau) = 0$) so by 10b) $\rho \equiv 0$. This however, is compatible with an arbitrary initial distribution of linear density $\rho(z, -\tau)$ = an arbitrary function of z. This determines the function $\phi'(x)$ for $0 < x < 1$ for (14a) becomes at time $-\tau$,

14e) $\phi'\left(\frac{z}{\ell+h}\right) = \phi'(x) = \frac{\ell+h}{\bar{m}}\, \rho\,(z,-\tau) = \frac{\ell+h}{\bar{m}}\, \rho((\ell+h)x, -\tau)$

where $x \equiv \frac{z}{\ell+h}$ so $z = (\ell+h)x$. When the initial density distribution is known this determines the function $\phi'(x)$. Whatever the initial density distribution it is evident from 14d that we may define the hydrodynamic mass m' of the powder by

15) $m' \equiv m + \frac{\bar{y}(-\tau)}{\ell + h}\,\bar{m}$ where $\bar{y}(-\tau)$ is the distance from breech to center of gravity of powder when the shell starts. Then since by (14d) $\dot{\bar{y}}(t) = \frac{\bar{y}(-\tau)}{\ell+h}\,\dot{y}(t)$ and $\ddot{\bar{y}}(t) = \frac{\bar{y}(-\tau)}{\ell+h}\,\ddot{y}(t)$ equation (5) becomes

16) $\frac{m'}{g}\,\ddot{y}\,(t) = \frac{M'}{g}\ddot{Y}(t) + D(t)$ for $-\tau \leq t$

If the powder chamber had everywhere the constant diameter of the bore, and if the initial density were uniform, $\frac{\bar{y}(-\tau)}{\ell+h}\,\bar{m}$ would be $\frac{\bar{m}}{2}$ which is Séberts approximation for the inertia of the powder. It is remarked by C. Cranz; Lehrbuch der Ballistik II, p. 379 that this approximation has generally been sufficient in practice. However, he considers only the case of a uniform tube. It seems that the observations of pressure, recoil, and velocities made by the Bureau of Standards are worth a more accurate evaluation. The ratio of cross section of powder chamber to bore is, in our gun, 1.4 so there is no justification for such a rough approximation. Moreover, there are indications that in the low-pressure rounds 4, and 7, the powder density may have been very non-uniform at the time the space was opened up as the shell began to move. The powder is only partially burned so that a denser or lighter fluid begins to follow the shell according as the center of gravity was initially nearer the shell or the breech.

In round (6) the charge $\bar{m}$ = 518 lbs is 2.2 times the charges 230 and 240 lbs of rounds (4) and (7). Hence more than half the powder chamber was empty before ignition of the latter charges.

In Hayes "Elements of Ordnance" p. 81, it is stated that "with small densities of loading, there is considerable variation in muzzle velocity, due probably to non-uniform ignition, and the fact that portions of the charge may be hurled against the base of the projectile and broken up. There sometimes results pressure singes or waves of considerable intensity." It would appear from this that the initial center of gravity might be near either end of the powder chamber. Hence in allowing for a possible non-uniformity of initial density we may find an explanation of the unpredictibility of behaviour of low-pressure rounds.

For rounds with powder charges not greatly different from the service charge for which the gun was designed, it is unreasonable to assume non-uniform volume density. Before working out this case it is easier to consider first the non-uniform, in which, since we do not know the distribution it is simpler to deal with linear instead of volume density. If the initial linear density $\rho(z,-\tau)$ is a linear function of z, then since the total charge is $\bar{m}$, it is found to be of the form, where B is some constant

17a) $\rho(z,-\tau) = \frac{\bar{m}}{\mathcal{L}+h}\left[1 - B + 2B\frac{z}{\mathcal{L}+h}\right]$ where $-1 < B < 1$

so that by (14e)

17b) $\phi'(x) = 1 - B + 2Bx$ so $\phi(x) = (1-B)x = Bx^2$ for $0 < x < 1$

hence equation (14a) becomes

17c) $\rho(z,t) = \frac{\bar{m}}{y}\left[1 - B + 2B\left(\frac{z}{y}\right)\right]$

and (14b) is

17d) $m(z,t) = \bar{m}\phi\left(\frac{z}{y}\right) = \bar{m}\frac{z}{y}\left[1 - B + B\frac{z}{y}\right]$

This gives for determining m' by (15)

17e) $\frac{\bar{y}(-\tau)}{\mathcal{L}+h} = \frac{1}{2}\left[1 + \frac{B}{3}\right]$ where $-1 < B < 1$

so that with the disposable constant B, the mass m' could be given values between $m' = m + \frac{\bar{m}}{3}$ and $m + \frac{2\bar{m}}{3}$ corresponding to the initial linear density (17a).

We consider from now on the more probable case where the initial volume density is uniform.

18a) $\bar{\rho}(z,-\tau) = \frac{\bar{m}}{V}$ = constant where V is the volume of the powder chamber which with b, c and h is numerically given in the designers' drawings and the effective length ℓ used here is thereby determined. The linear density is

18b) $\rho(z,-\tau) = \frac{\bar{m}}{V}\pi r^2(z)$ (for initial uniform volume density)

Reference to fig. 1 shows

$$18c)\quad \begin{aligned} r(z) &= b \text{ ------------------------ if } 0 < z < \ell \\ &= c\left[1 + \epsilon\left(\frac{\ell+h}{h}\right)\left(1 - \frac{z}{\ell+h}\right)\right] \text{ if } \ell \leq z \leq \ell + h \\ &= c \text{ ------------------------ if } \ell + h \leq z \end{aligned}$$

18d) $\epsilon \equiv \frac{b-c}{c} = .1786,\ \epsilon^2 = .0320 \quad \frac{h}{\ell} = .2690,\ \frac{h^2}{\ell^2} = .0724$

$$18f)\quad V = \pi b^2\ell + \frac{\pi h}{3}(b^2+bc+c^2) = $$

$$= \pi c^2\ell\left[1 + 2\epsilon + \epsilon^2 + \frac{h}{\ell}\left(1 + \epsilon + \frac{\epsilon^2}{3}\right)\right] = \pi c^2\ell(1.709) = 11.316 \text{ cu. ft.}$$

and

$$18g)\quad \bar{y}(-\tau) = \frac{\pi c^2}{V}\left[\frac{(\ell+h)^2}{2} + \epsilon\left(\ell^2+\ell h+\frac{h^2}{3}\right) + \frac{\epsilon^2}{2}\left(\ell^2+\frac{2\ell h}{3}+\frac{h^2}{6}\right)\right] = 3.830 \text{ ft.}$$

so that

$$18h)\quad \frac{y(-\tau)}{\ell+h} = \frac{\left(1+\frac{h}{\ell}\right)^2 + 2\epsilon\left(1+\frac{h}{\ell}+\frac{h^2}{3\ell^2}\right) + \epsilon^2\left(1+\frac{2h}{3\ell}+\frac{h^2}{6\ell^2}\right)}{2\left(1+\frac{h}{\ell}\right)\left[1 + 2\epsilon^2 + \frac{h}{\ell}\left(1 + \epsilon + \frac{\epsilon^2}{3}\right)\right]} = .487_1$$

On the hypothesis of initial uniform volume density which we adopt in general, this equation (18h) is to be used in (15) for the mass m'. If we wish to consider cases of non-uniform initial density of type (17a) then (17e) is to be used in (15) for m'.

For the uniform initial volume density which is considered, unless otherwise stated, we find by (14e) and (18b)

$$\phi'\left(\frac{z}{\ell+h}\right) = \phi'(x) = \frac{\ell+h}{V}\pi r^2(z) \quad \text{where } x = \frac{z}{\ell+h}$$

or by (18c)

19a) $\phi'(x) = \left(\frac{\ell+h}{V}\right) \pi b^2$ ------------------ when $0 \leq x \leq \frac{\ell}{\ell+h}$

19b) $\phi'(x) = \frac{(\ell+h)\pi c^2}{V} \left[1 + \epsilon\left(\frac{\ell+h}{h}\right)(1-x)\right]^2$ -- when $\frac{\ell}{\ell+h} \leq x \leq 1$

Integrating this gives since $\phi(0) = 0$ and $\phi(1) = 1$

20a) $\phi(x) = \left(\frac{\ell+h}{V}\right)\pi b^2 x$ ---------------- when $0 < x \leq \frac{\ell}{\ell+h}$

20b) $$\phi(x) = \frac{(\ell+h)\pi}{V}\left\{b^2x - \frac{b(b-c)}{h}(\ell+h)\left(x-\frac{\ell}{\ell+h}\right)^2 + \frac{(b-c)}{3h^2}(\ell+h)^2 \cdot \left(x-\frac{\ell}{\ell+h}\right)^3\right\} = 1 - \frac{\pi c^2(1-x)}{V}\left\{1 + \frac{\epsilon(\ell+h)}{h}(1-x) + \frac{1}{3}\left[\epsilon\left(\frac{\ell+h}{h}\right)(1-x)\right]^2\right\}$$

if $\frac{\ell}{\ell+h} \leq x \leq 1$

Using the expressions (19a) and (19b) in the general formula (14a) the explicit expression for the linear density as a function of z and y becomes

21) $\rho(z,t) = \frac{\bar{m}(\ell+h)\pi b^2}{V\,y(t)}$ when $0 \leq z \leq \frac{\ell}{\ell+h} y(t) = .788\ y$

$= \frac{\bar{m}(\ell+h)\pi c^2}{V\,y(t)} \left[1 + \epsilon\frac{(\ell+h)}{h}\left(1 - \frac{z}{y}\right)\right]^2$ when $\frac{\ell}{\ell+h} y(t) \leq z \leq y(t)$

The instantaneous pattern of density distribution may be pictured by considering the moving plane

$z = \frac{\ell}{\ell+h} y(t) = .788\ y(t)$ which is the boundary of a region of uniform linear density on the left and one of decreasing linear density on the right which decreases until at the base of the shell its value is $\frac{\ell^2}{b^2}$ times (or .72 times) its value at the breech.

It is of more interest to picture the volume density, $\rho_v(z,t) = \frac{\rho(z,t)}{\pi r^2(z)}$, which is always uniform in the cylindrial part of the powder chamber but has a maximum value and decreases until at the base of the shell it has returned to the value at the breech. There is a small transition interval

of time in which the shell travels a distance
$y - l - h = h\left(1 + \frac{h}{\ell}\right) = 2.11$ feet during which the volume density in the conical region changes from uniform to a final distribution which remains relatively the same thereafter, then falling off uniformly with time due to the increasing y which is a factor of the denominator. The point $z = \ell + h$ always has the maximum volume density which is a point maximum during the first interval. When $y = h\left(1+\frac{h}{\ell}\right)$ this maximum volume density becomes 39 percent higher than at the breech. After the shell has moved more than 2.13 the maximum value extends from $z = \ell + h$ to $z = \frac{\ell y}{\ell + h}$.
Beyond this point the volume density decreases toward the shell reaching there the value at the breech. If we let $C = \frac{\bar{m}(\ell+h)}{V}$, the expressions for the volume density are

<u>At a time in the transition interval</u> $0 \leqslant y - \ell - h \leqslant h\left(1 + \frac{h}{\ell}\right) = 2.11$ ft. the volume density is

22a) $$\frac{y}{C}\rho_v(z,t) = 1 \text{ ------------ if } 0 \leqslant z \leqslant \ell \quad \text{cylindrical}$$

$$\left.\begin{aligned} &= \frac{b^2}{c^2\left[1+\epsilon\left(\frac{\ell+h}{h}\right)\left(1-\frac{z}{\ell+h}\right)\right]^2} \quad \text{if } \ell \leqslant z \leqslant \frac{\ell y}{\ell+h} \\ &= \left[\frac{1+\epsilon\left(\frac{\ell+h}{h}\right)\left(1-\frac{z}{y}\right)}{1+\epsilon\left(\frac{\ell+h}{h}\right)\left(1-\frac{z}{\ell+h}\right)}\right]^2 \quad \text{if } \frac{\ell y}{\ell+h} \leqslant z \leqslant \ell + h \end{aligned}\right\} \text{conical}$$

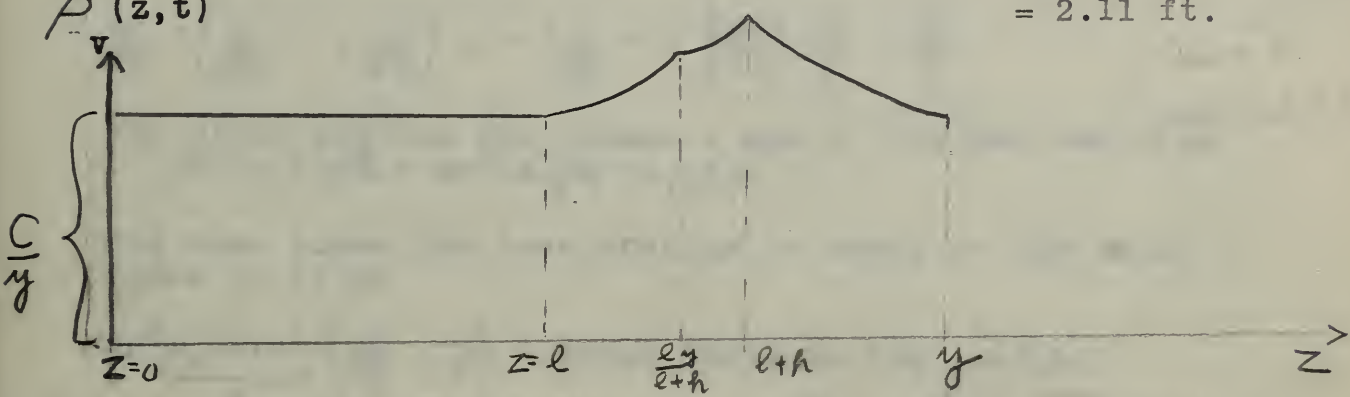
$$= \left[1 + \epsilon\left(\frac{\ell+h}{h}\right)\left(1-\frac{z}{y}\right)\right]^2 \quad \text{if } \ell + h \leqslant z \leqslant y \quad \text{(bore)}$$

<u>At a time in the remaining interval</u> $y - \ell - h = 2.11$ to ejection the density is

22b) $$\left.\begin{aligned} y\,\frac{\rho_v(z,t)}{C} &= 1 \quad \text{if } 0 \leqslant z \leqslant \ell \quad \text{Cylindrical} \\ &= \frac{b^2}{c^2}\,\frac{1}{\left[1+\epsilon\left(\frac{\ell+h}{h}\right)\left(1-\frac{z}{\ell+h}\right)\right]^2} \quad \text{if } \ell \leqslant z \leqslant \ell + h \quad \text{Conical} \end{aligned}\right\} \text{Powder Chamber}$$

$$\left.\begin{array}{ll} = \dfrac{b^2}{c^2} \text{ ----------------------- } & \text{if } \ell+h \leqslant z \leqslant \dfrac{\ell y}{\ell+h} \\ = \left[1 + \epsilon\left(\dfrac{\ell+h}{h}\right)\left(1-\dfrac{z}{y}\right)\right]^2 & \text{if } \dfrac{\ell y}{\ell+h} \leqslant z \leqslant y \end{array}\right\} \text{(bore)}$$

Since $\epsilon = \frac{b-c}{c}$ it is evident that the density is a continuous function of z and of the time, but its z-derivative (although always finite) has finite discontinuities so the point-maximum (when it exists) of the curve plotting d against z for any instant t is a cusp. After the transition period, this maximum value has spread through most of the bore behind the shell. The instantaneous curves for volume density d against z are sketched for the two cases. Case 1 $y-\ell-h < h(1+\frac{h}{\ell})$ = 2.11 ft.



(Fig.2_a ·Volume density at a time in the transition interval

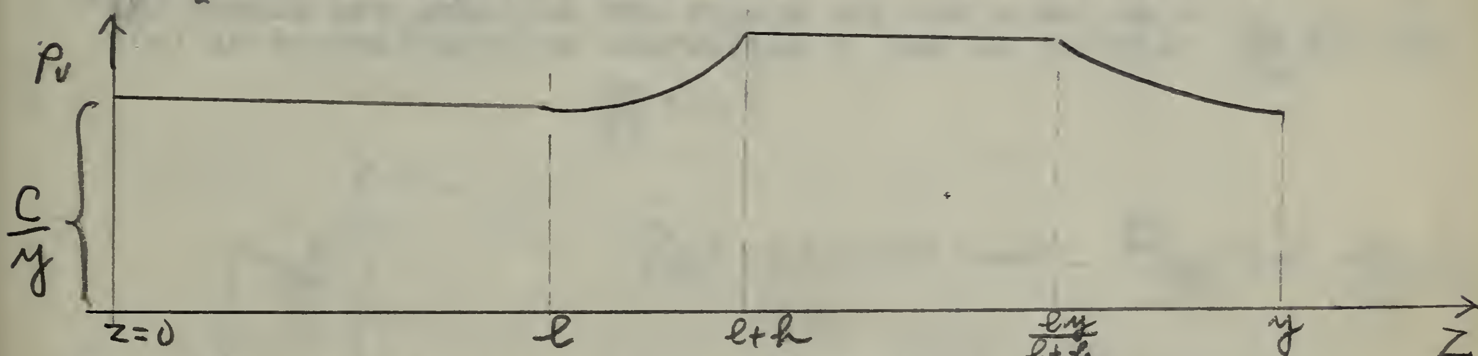


Fig. 2_b Volume density at a time after the transition interval.

In this curve the moulding effect of the shape of the powder chamber is still discernable although the initial pattern of density is elongated as the fluid expands. The case where the initial volume density ρ_v is a linear function of z instead of uniform could be found and would be more complicated,

but when the linear density ρ is a linear function of z as in (17a) the later density ρ given by (17c) is much simpler.

IV. Space Variation of Pressure and Pressure on the Shell.

The relative acceleration of the fluid is the total time derivative $\frac{du}{dt}$ of its relative velocity, that is the time rate of increase of relative velocity u as it would be reckoned moving with the fluid as distinguished from its rate of increase $\frac{\partial u}{\partial t}$ reckoned at a fixed point. This relative acceleration is

$$\frac{du}{dt} = \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial z}\right) u = \left(\frac{\partial}{\partial t} + z\frac{\dot{y}\,\partial}{y\,\partial z}\right)\frac{z\dot{y}}{y} = \frac{z\ddot{y}}{y}$$

The fluid between the planes z and z + dz has the mass

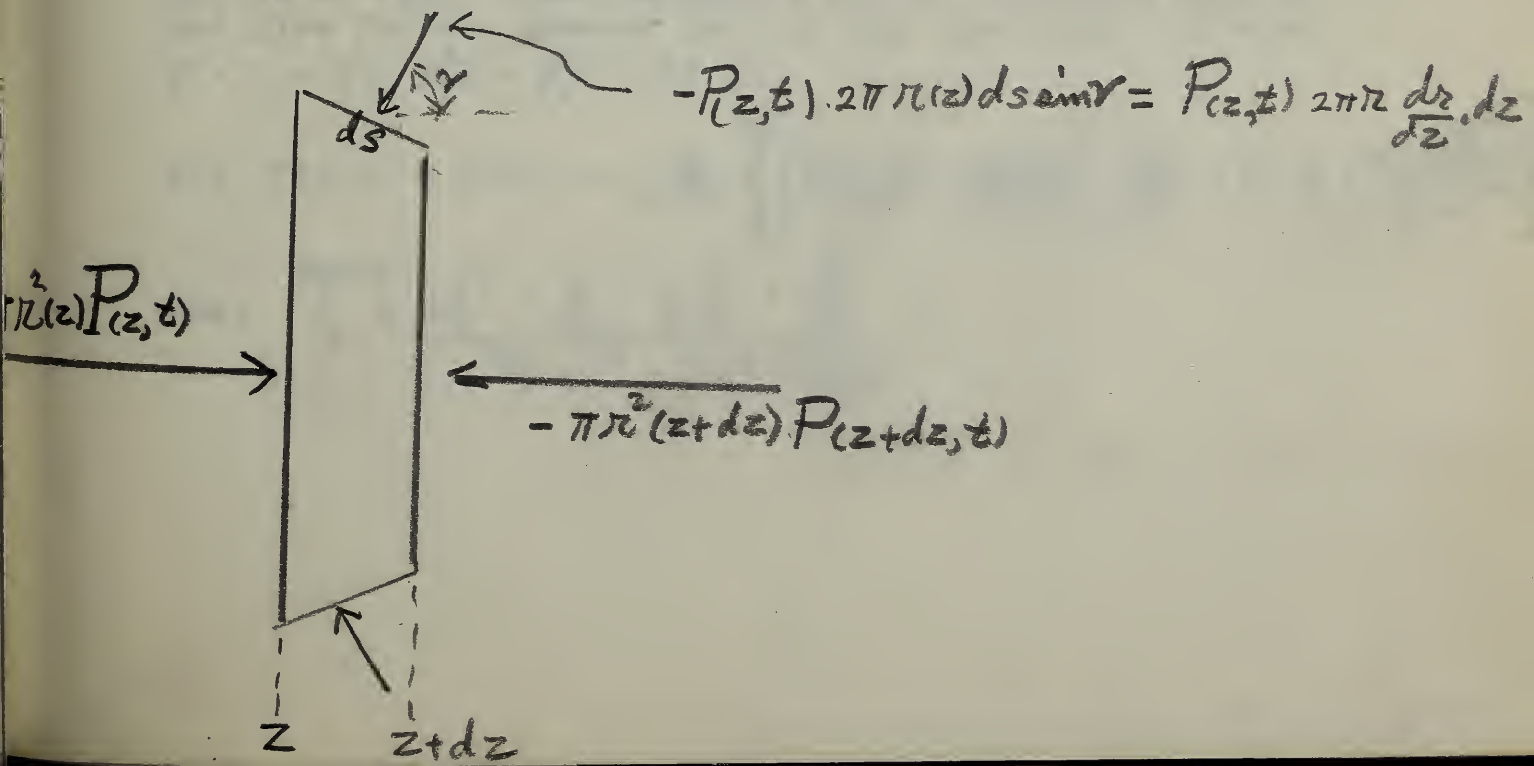
$$\rho(z,t)dz = \pi r^2(z)\rho_v(z,t)dz$$

Its mass times its acceleration is equal to the total force on it or

$$\frac{\pi r^2\rho_v(z,t)}{g}\left[\frac{z\ddot{y}}{y} - \ddot{y}\right] = \text{total force in lbs. weight}$$

The forces are shown in the figure for the case where r(z) is decreasing with increasing z, and ds sin $\gamma = -\frac{dr}{dz}dz > 0$

$$-\frac{dr}{dz}dz > 0$$



The pressure at a point on the boundary surface of the powder chamber is here considered equal to the mean pressure over the circular section passing through that point. Hence the equation of mean fluid motion is

$$\pi r^2(z)\frac{\rho(z,t)}{g}\left[\frac{z\ddot{y}}{y}-\ddot{Y}\right] = -\pi\frac{\partial}{\partial z}\left(r^2(z)P(z,t)\right) +$$

$$2\pi r(z)\frac{dr}{dz}P(z,t) = -\pi r^2(z)\frac{\partial P(z,t)}{\partial z}$$

that is

$$23)\quad -\frac{\partial P(z,t)}{\partial z} = \frac{\rho_v(z,t)}{g}\left[z\frac{\ddot{y}}{y}-\ddot{Y}\right] \text{ where } \rho_v = \frac{\rho}{\pi r^2} =$$

the volume density. Integrating this gives

$$24)\quad P(z,t) - P(o,t) = -\frac{1}{g}\int_o^z \rho_v(z,t)\left[z_1\frac{\ddot{y}}{y}-\ddot{Y}\right]dz_1.$$

To integrate this when the initial volume density is uniform, there are two cases to consider, and in each case the explicit expression for $\rho_v(z,t)$ is given by four different expressions in (22a) or (22b), by which (24) may be integrated as soon as the point z is chosen. The curves of fig 2a and fig. 2b show in which of the four regions the upper limit z lies. The pressure on the shell is

$$24)'\quad P(y,t) - P(o,t) = -\frac{1}{g}\int_o^y\left(z\frac{\ddot{y}}{y}-\ddot{Y}\right)\rho_v(z,t)\,dz$$

$$= -\frac{\overline{m}}{gV}\left(\frac{\ell+h}{y}\right)\left[\frac{\ddot{y}\ell^2}{2y}-\ddot{Y}\ell+\int_\ell^{\ell+h}\left[\frac{z\ddot{y}}{y}-\ddot{Y}\right]dz+\int_{\ell+h}^{y}\left[\frac{z\ddot{y}}{y}-\ddot{Y}\right]\left(\frac{y\rho_v}{c}\right)dz\right]$$

By use of (22b) these integrals may be evaluated exactly and give for the greater part of the time, that is, when $y - l - h \geqslant h\left(1+\frac{h}{\ell}\right) = 2.13$ ft.

$$25)\quad P(y,t) = P(o,t) - \frac{\overline{m}}{\pi c^2 g}\left\{\left[\frac{y(-\tau)}{\ell+h}-\beta_o\left(\frac{\ell+h}{y}\right)^2\right]\ddot{y}-\left[1-\beta_o\left(\frac{\ell+h}{y}\right)\right]\ddot{Y}\right\}$$

where

$$25a)\quad \beta_o = \frac{2\epsilon\left[1+\frac{h}{2\ell}+\frac{\epsilon}{2}\left(1+\frac{h}{\ell}\right)\right]}{1+2\epsilon+\epsilon^2+\frac{h}{\ell}\left(1+\epsilon+\frac{\epsilon^2}{3}\right)}$$

and

$$25b)\quad \beta = \epsilon \frac{\left\{1 + \epsilon\left(\frac{\ell+h}{2\ell}\right) + \frac{h^2}{2\ell(\ell+h)\epsilon}\left(\frac{2(1+\epsilon)\left[(1+\epsilon)\log(1+\epsilon)-\epsilon\right]}{\epsilon^2} - 1\right)\right\}}{1 + 2\epsilon + \epsilon^2 + \frac{h}{\ell}\left(1 + \epsilon + \frac{\epsilon^2}{3}\right)}$$

$$= \epsilon \frac{1 + \epsilon\left(\frac{\ell+h}{2\ell}\right) + \frac{h^2}{3\ell(\ell+h)}\left(1 - \frac{\epsilon}{4} + \frac{\epsilon^2}{10} --- \right)}{1 + 2\epsilon + \epsilon^2 + \frac{h}{\ell}\left(1 + \epsilon + \frac{\epsilon^2}{3}\right)} \cong \epsilon\, \frac{1.133}{1.718}$$

The numerical values are

$$25c)\quad \beta = .118_3 \qquad \text{and} \qquad \beta_o = .261$$

Although (25) is only exact for $y - \ell - h \geqslant h(1 + \frac{h}{\ell}) = 2.11$ ft., it is sufficiently accurate for all values of y, as shown by the fact that when $y - \ell - h$ is very small compared to $\ell + h$ (so that $\ddot{y}$ is very small and $\ddot{Y}$ very much smaller) equation (25) gives

$P(y,t) = P(o,t) - \frac{\bar{m}}{\pi c^2 g}(.369)\ddot{y}$ whereas the exact value found by using (22a) and (24)' is practically the same. It is

$$25d)\quad P(y,t) = P(o,t) - \frac{\bar{m}}{\pi c^2 g}\; \frac{(1 + \frac{h}{\ell})\ddot{y}}{2\left[1 + 2\epsilon + \epsilon^2 + \frac{h}{\ell}\left(1 + \epsilon + \frac{\epsilon^2}{3}\right)\right]}$$

$$= P(o,t) - \frac{\bar{m}}{\pi c^2 g}(.371)\ddot{y}$$

V. The Frictional Force F(t) on the Shell

If we use (25) for P(y,t) in the equation (2) of motion of the shell, this gives the frictional force on the shell as the difference between two large terms, each of which is at present determined with a large error. The result is

$$26a)\quad F(t) = \pi c^2 P(o,t) - m'\frac{\ddot{y}}{g}\left[1 - \frac{\bar{m}}{m'}\cdot\beta\left(\frac{\ell+h}{y}\right)^2\right] + \frac{\ddot{Y}}{g}\left[m + \bar{m}\cdot\left(1 - \beta_o\left(\frac{\ell+h}{y}\right)\right)\right]$$

If $\ddot{y}$ and $\ddot{Y}$ are both known, this determines F(t) in terms of the reading of the pressure gage.
By use of (16) this may be written

$$26b)\quad F(t) = \pi c^2 P(o,t) - \left[1 - \frac{\bar{m}}{m'}\beta\left(\frac{\ell+h}{y}\right)^2\right]\cdot\left[\frac{M'\ddot{Y}(t)}{g} + D(t)\right] + \frac{\ddot{Y}}{g}\left\{m + \bar{m}\left[1 - \beta_o\left(\frac{\ell+h}{y}\right)\right]\right\}$$

This is an exact derivation from the theory. In service rounds the factor $\frac{\bar{m}}{m'}\beta$ is .035 but this term is not in general negligible although it is small compared to unity because when the acceleration of recoil is large the difference

$\pi c^2 P(o,t) - \left[\frac{M'\ddot{Y}}{g} + D\right]$ may be small compared to $\left[\frac{M\ddot{Y}}{g} + D\right]$.

At time $t \rightarrow +0$, $\ddot{Y}$ is zero and $\frac{\ell+h}{y}$ nearly unity so that equation 26b) becomes

$$26c)\quad F(+o) + \pi c^2 P_o - \left(1 - \beta\frac{\bar{m}}{m'}\right) D(+o) \quad \text{where } P_o = P(o,o)$$

In table 1 are shown the force F(+o) computed by this equation where, as will be shown later,

26d) D(+o) = 73,000 pounds weight for all rounds. The time rate of rise of pressure $\dot{P}_o \equiv \dot{P}(o,o)$ is also shown. It could be determined from the pressure-time curves with an estimated error of ±30 percent for all rounds except the two low pressure rounds 4 and 7 where the error may be double this. The values of P_o may be in error by ±50 percent in general or double this for the low-pressure rounds. The force D(+o) is considered known to ± five percent.

Table 1, Pressures and Friction at time t=o.

Round	Charge $\bar{m}$ lbs.	m' lbs.	$P_o \frac{lbs.}{in^2}$	$\dot{P}_o \frac{lbs.}{in^2 sec}$	F(+o)lbs.
1	484	1,636	4,500	$2.2(10)^6$	$.62\ 10^6$
2	484	1,636	6,000	3.0 "	.85 "
3	484	1,636	7,500	2.5 "	1.08 "
4	230	1,512	---	.5 "	---
5	368	1,578	10,000	1.1 "	1.47 "
6	518	1,652	4,500	6.0 "	.62 "
7	240	1,517	---	.55"	---

This determination of $F(+o)$ is as accurate as the determination of pressure at time $t = 0$ since no errors in $\ddot{Y}$ are of importance. Again near ejection the acceleration of recoil $\ddot{Y}$ is relatively small and the term $\frac{M'}{g}\ddot{Y} + D$ is appreciably less than the pressure term $\pi c^2 P(o,t)$.

Hence, at times near ejection an evaluation of F by (26b) might rank next in accuracy to the evaluation at start of recoil.

When the difference between the two large terms in (26b) is small compared to either, the determination of F by this equation will be practically impossible as the small percentage errors in pressure and acceleration will be the principal part of the second equation. Large fluctuations will mask the value we are seeking and even give the absurdity of negative F.

Since it may be necessary to resort to some other method of determining F, it is worth while to consider its origin in detail, assuming the engraving is completed.

In the motion of the gun, the kinetic friction between the gun and its slide was considered negligible compared to static friction since the greatest velocity was about 20 ft. per sec. The shell attains a velocity about one hundred fifty times this or 3000 ft. per sec. so that kinetic friction may be an appreciable part of the force F. This may be assumed to be represented by a term $F_1 = \nu_1 y^k$ where ν_1 and k are positive constants to be found. This term may be considered as representing kinetic friction contributed by all the surface of the shell except the rotating band whose interaction with the lands must be considered separately.

A second force F_a which might possibly be appreciable is the resistance of the air which does not have time to flow out of the barrel but is pushed out, the shell gathering it up as it proceeds, so that it adds an ever increasing inertia.

Since the initial force $F(+o)$ is of the order of $(10)^6$ lbs., we may consider parts of it of the order .02 $(10)^6$ lbs. as negligible. The atmospheric pressure on the shell amounts to about one-tenth of this or .002 $(10)^6$, but the inertial effect is much larger. The shell imparts its own acceleration $\ddot{y}$ to the total mass of air it has gathered up which is $\rho\pi c^2(y-l-h)$ lbs. ~~In addition, the shell picks up in-~~

$= .086(y-l-h)$. This requires a force

$\frac{\rho\pi c^2}{g}(y-l-h)\ddot{y}$ lbs. In addition the shell picks up in

time dt a resting mass of air $\rho\pi c^2\dot{y}$ dt to which it imparts the velocity $\dot{y}$ so the time rate of change of momentum is $\rho\pi c^2\dot{y}^2$. Hence the inertial effect of the air gives a retarding force on the shell of

$$F_a(t) = \frac{\rho\pi c^2}{g}\left[(y-\ell-h)\ddot{y}+\dot{y}^2\right] \text{lbs} = \frac{\rho\pi c^2}{g}\frac{d}{dt}\left[(y-\ell-h)\dot{y}\right]$$

$$= .00269\left[(y-\ell-h)\ddot{y}+\dot{y}^2\right]$$

In tables 9 and (10) below $y-\ell-h$, $\ddot{y}$ and $\dot{y}$ are given for round 2 and these show that $F_a = .011\ (10)^6$ (at $t = .020$ sec), $= .025\ (10)^6$ lbs (at $t = .0315$ sec). Hence, we conclude that the effect of the air is negligible.

An important part of F is what may be called the elastic force F_e. After the engraving, when the shell has been pushed into the barrel, both it and the barrel will be under a large elastic compression and both slightly deformed. There will be a retarding force on the shell like that on a tight fitting cork which is pushed through a bottle. The barrel is expanded behind the shell due to the powder pressure and also to the expansive force of the compressed shell, so that in front of the rotating band, the barrel will be constricted. The effect of decreasing powder pressure will be to decrease this force as the shell travels, but the effect of decreasing thickness of the gun's walls will tend to modify this force. Also as the shell moves into parts where erosion has increased the bore, the force will be decreased. When the shell nears the muzzle, the powder pressure is slowly decreasing but the gun's walls begin to increase in thickness so that there may be a choking effect represented by a rise in the force F_e as the shell approaches ejection.

A fourth force F_4 comes from the action of the driving edge of the rifling upon the grooves of the rotating band. The rifling angle is γ where $\tan\gamma = \frac{1}{32}$ since the rifling has a uniform twist of one in 32 calibers. The action of the lands is illustrated in the accompanying figures, where in fig. a the circle P_oA is a section of the bore of radius $c = 7/12$ ft., and oy is the gun's axis while the curve P_oP is the forward or driving edge of the lands. In fig. b the inner barrel surface is developed or rolled out on a plane

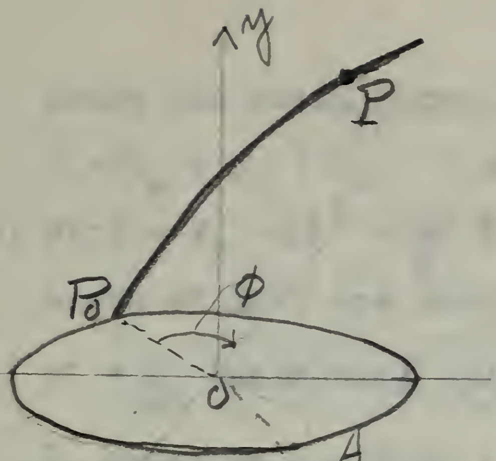


Fig a

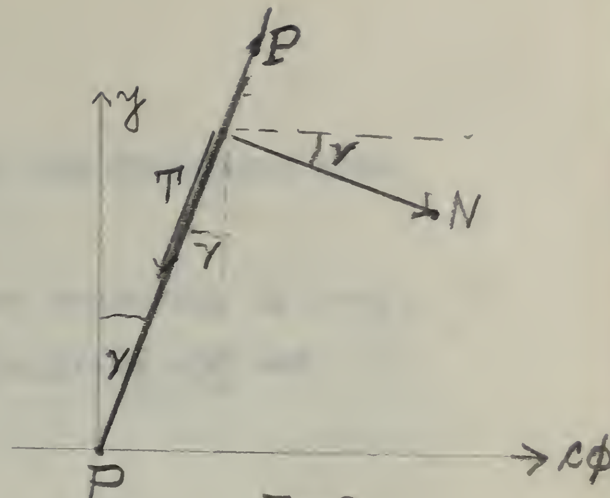


Fig b

The total force of the lands on the engraving bands has component N normal to the curve and tangential component T. The part of T which represents static friction is $\mu_0 N$. The part representing kinetic friction is ν_0 times the Kth power of the relative velocity v_t of the shell in the tangential direction. The angular velocity of the shell being $\dot{\phi}$, where

27a) $c\dot{\phi} = \tan\gamma\, \dot{y}$,

then the linear velocity of points on the rotating band in the tangential direction is $v_t = \dfrac{\dot{y}}{\cos\gamma} = \dot{y}$ practically since γ is small.
Hence we assume

27b) $T = \mu_0 N + \nu_0 \dot{y}^k$

Resolving T and N along the axis gives

27c) $F_4 = N\sin\gamma + T\cos\gamma = N(\sin\gamma + \mu\cos\gamma) + \nu_0 \cos\gamma\, \dot{y}^k$

The equation of angular motion of the shell is(if I is the moment of inertia of the shell about its axis)

27d) $\dfrac{I\ddot{\phi}}{gc} = \dfrac{I\tan\gamma}{gc^2}\ddot{y} = (N\cos\gamma - T\sin\gamma)$

Replacing T by its value (27b) this becomes

27e) $N = \dfrac{1}{\cos\gamma - \mu_0 \sin\gamma}\left[\dfrac{I\tan\gamma}{gc^2}\ddot{y} + \nu_0 \sin\gamma\, \dot{y}^K\right]$

Using this value of N in (27c) gives

$$F_4 = \frac{1}{1-\mu_0\tan\gamma}\left[\frac{I\tan\gamma}{c^2 g}(\tan\gamma+\mu_0)\ddot{y} + \frac{\nu_0\dot{y}^k}{\cos\gamma}\right]$$

Or since γ is small

27f) $$F_4 = \frac{I\tan\gamma(\tan\gamma+\mu_0)}{c^2 g}\ddot{y} + \nu_0\dot{y}^K$$

Hence the total force F would be of the following form

$F = F_e + F_1 + F_4$ or

27g) $F(t) = F_e + \nu \dot{y}^k + \frac{m''}{g} \ddot{y}$ pounds (after engraving is over)

where m'', ν, k are positive constants, $\nu = \nu_0 + \nu_1$ and

27h) $m'' = \frac{I}{c^2} \tan\gamma(\tan\gamma + \mu_0)$

$\frac{I}{c^2}$ would be $\frac{m}{2} = \frac{1400}{2} = 700$ if the shell were cylindrical.

It is somewhat less, varying from 650 to 660. so $\frac{I}{c^2} = .47$ m
Also $\tan\gamma = \frac{1}{32}$

The first term in (27f) has the effect of loading the shell with the extra mass m".

We may take $\mu_0 = \frac{1}{2}$ as an upper limit for the static friction at the rifling (after the engraving is completed). This gives

$$m'' = m \times \frac{.47}{32}\left(.5 + \frac{1}{32}\right) = .0078\ m = .0078 \times 1400 = 11 \text{ pounds}$$

Hence the force, $\frac{m'}{g}\ddot{y} = \frac{11\ddot{y}}{g} = .34\ \ddot{y}$, has a maximum value $.042(10)^6$ lbs. corresponding to the greatest acceleration $\ddot{y}$ given in table 10 below, $\ddot{y} = .125(10)^6$ ft/sec^2 for round 2.

This indicates that the third term in (27g) is always negligible, as would be expected since it amounts to increasing the shell's mass of 1400 lbs. by 11 pounds.

The conclusion is that the force must be of the form

28) $F(t) = F_e + \nu \dot{y}^k$

where F_e is the elastic force and $\nu \dot{y}^k$ that of kinetic friction at all parts of the shell.

VI. The Starting Force F_s on the Shell and the Time When it Starts.

In the interval $-\tau \leqslant t \leqslant 0$ the pressure-time curves are nearly linear so that

29a) $P(o,t) = P_o + t\dot{P}_o$ for $-\tau \leqslant t \leqslant 0$

Since $\ddot{y} = \ddot{Y} = 0$ when $t = -\tau$ the starting force $F_s = F(-\tau)$ is by (26a) and (29a)

$$29b)\quad F_s = \pi c^2 P_o - \tau \pi c^2 \dot{P}_o$$

Also since $\ddot{Y} \equiv 0$ in the interval $-\tau \leqslant t \leqslant 0$ and $\frac{l+h}{y}$ is practically equal to one, the equation (26a) gives the equation of motion of the shell in the following form by use of (29a) and (29b)

$$29c)\quad \left(1-\beta\frac{\overline{m}}{m'}\right)\frac{m'}{g}\ddot{y}(t) = (t+\tau)\,\pi c^2\dot{P}_o + F_s - F(t) \quad \text{for } -\tau \leqslant t \leqslant o$$

while equation (16) gives its equation of motion through the same interval in the form

$$29d)\quad \frac{m'}{g}\ddot{y}(t) = D(t) \text{ --------- } \quad \text{for } -\tau \leqslant t \leqslant o$$

From these two equivalent forms, the external force in this interval is connected with F by the relation

$$30)\quad F(t) = F_s + (t+\tau)\,\pi c^2\dot{P}_o - \left(1-\beta\frac{\overline{m}}{m'}\right) D(t)$$
$$= \pi c^2 P_o + t\pi c^2 \dot{P}_o - \left(1-\beta\frac{\overline{m}}{m'}\right) D(t)$$

Letting $t \rightarrow -0$ in this gives

$$31)\quad F(-o) = \pi c^2 P_o - \left(1-\beta\frac{\overline{m}}{m'}\right) D(-0)$$

so that by (26a)

$$32)\quad \left(1-\beta\frac{\overline{m}}{m'}\right)\left[D(-0) - D(+0)\right] = F(+0) - F(-0)$$

Hence if the external force D has a finite discontinuity at t = 0, it is accompanied by a discontinuity in the force exerted by the gun on the shell. Such a discontinuity was indicated during the observations of the compressive force of the recoil springs and the friction between the gun and its slide. From the time the shell starts to an appreciable time after the gun recoils, the only external forces acting on the gun are the constant force Ks exerted by the recoil springs on account of a considerable initial compression s, the constant component of the gun's weight, the friction between gun and slides, and, before recoil, the back reaction S(t) of the slide carriage where it presses against the yoke,

33) $D(t) = Ks - M \sin\theta + \mu(t) M \cos\theta - S(t)$ lbs weight

where $\theta = 8° =$ the elevation. The resistance $S(t)$ to the pressure of the spring can never be negative.

Before the shell moves, the excess of spring force over component of weight $Ks - M\sin\theta$ is balanced by the force $\mu(t)M' \cos\theta - S(t)$ but the manner in which this force is shared between friction $\mu(t)M\cos\theta$ and S is unknown and depends upon the manner in which the gun had been previously brought to battery.
For $t \leqslant -\tau$, $D(t) = 0$ and as t increases from $-\tau$ to -0 $D(t)$ rises from zero to a value $D(-o)$. Shortly before $t = 0$ the resistance S ceases to act, causing μ to rise to its greatest possible value $\mu(-0) = \mu(+0) = .047(1 \pm .05)$ ~~.05~~ = the coefficient of starting friction (and presumably of sliding friction). This was found by a series of observations in which the gun was elevated very slowly by hand and the successive angles θ_1, θ_2, θ_3 --- noted at which it began to slide gently out of battery. From three such observations, three independent equations of the form $Ks - M\sin\theta_n + \mu(+0)M\sin\theta_n = 0$ $n = 1,2,3$ -- were obtained, whose solution gave (taking $M = 206{,}000$ lbs.)

33a) $\begin{cases} Ks = 91{,}500 \text{ lbs. weight and } \mu(+0)M = 9{,}600 \text{ lbs. wt.} \\ K = 2{,}100 \text{ lbs. weight per inch compression} = 25{,}200 \\ \text{lbs. per foot, which give } \mu(+0) = .047 \text{ and } s = 3.63 \text{ ft.} \end{cases}$

After the seven rounds were fired, these measurements were repeated as a check. The values of Ks, K and μ were considered to be known to ±5 percent of their values.

In service conditions, the gun after firing is restored to battery with some velocity and it was found to stick or become wedged into the slide as shown by the fact that in this case values of μ were obtained which were three times $\mu(+0)$. As sticking of the gun is an erratic thing, it is safe to take $\mu(-0) = .2 \pm .15 = .2(1 \pm 3/4)$

hence

33b) $D(+0) = Ks - Mg\sin\theta + \mu(+0)M\cos\theta = 73{,}000$ lbs. wt.

33c) $D(-0) = Ks - Mg\sin\theta + \mu(-0)M\cos\theta = 104{,}000$ lbs wt.

where $\theta - 8°$, $\mu(+0) = .047$ and $\mu(-0) = +.2(1 \pm 3/4)$.

The allowance of ±75 percent variation in $\mu(-0)$ from round to round on account of unequal sticking of the gun only makes a variation of ±30 percent in the value of D(-0), and the extreme (-30 percent) makes D(-0) practically the same as D(+0). As the gun begins to move, the sticking disappears so that in all cases the lower coefficient $\mu(+0)$ is supposed to apply after recoil begins, where D(+0) is considered known to ±5 percent of itself.

During the small time that elapses between the start of the shell and of the gun, we may assume that the frictional force F(t) between gun and shell is practically constant

34) $$F(t) = F_s = F(-\tau) \quad \text{for } -\tau \leqslant t < 0$$

so that equation (30) becomes

35) $$D(t) = \frac{\pi c^2 \dot{P}_o}{1-\beta\frac{\overline{m}}{m'}}(t+\tau) \text{ --- for } -\tau \leqslant t < 0$$

and equation (29d) becomes

36) $$\frac{m'}{g}\ddot{y}(t) = D(t) = D(-0)\left(\frac{t+\tau}{\tau}\right) = \frac{\pi c^2 \dot{P}_o}{1-\beta\frac{\overline{m}}{m'}}(t+\tau) \text{ for } -\tau \leqslant t < 0$$

Placing t = -0 in (35) gives

37a) $$\tau = \left(1 - \beta\frac{\overline{m}}{m'}\right)\frac{D(-0)}{\pi c^2 \dot{P}_o} = \left(1 - .118\frac{\overline{m}}{m'}\right)\frac{.104(10)^6}{\pi c^2 \dot{P}_o}$$

which with (31) gives the force F_s necessary to start the shell

37b) $$F_s = \pi c^2 P_o - \left(1 - \beta\frac{\overline{m}}{m'}\right) D(-o)$$

$$= F(+o) - \left(1 - \beta\frac{\overline{m}}{m'}\right)\left[D(-o) - D(+o)\right] \text{ ---- by (26c)}$$

$$= F(+o) - \left(1 - .118\frac{\overline{m}}{m'}\right) \times 31{,}000 \text{ lbs wt. by (33b)}$$

and (33c).

Using the figures given in Table 1, we find the following values for starting force F_s and time interval τ by equation (37)

Table 2 - Starting force F_s on shell and time τ

Round	F_s	τ
1	$.6(10)^6$ lbs. wt.	.00030 sec
2	$.8(10)^6$ " "	.00020 "
3	$1.1(10)^6$ " "	.00026 "
4	-----	.00130 "
5	$1.4(10)^6$ " "	.00060 "
6	$.8(10)^6$ " "	.00011 "
7	-----	.00120 "
mean	$1(10)^6$ " "	

The time τ varies greatly with charge but the average variation of starting force from the mean $F_s = (10)^6$ lbs. is within the estimated error ±30 percent. It is practically the same as F(+o) given in Table 1. This corresponds to a pressure of 6,500 lbs./in^2 which compares favorably with the figure 8,000 lbs./in^2 rumored to have been found by the Army Department, by pushing the shell through the barrel.

The estimates of $\dot{P}_o$ for the low pressure rounds 4 and 7 are rough and the relatively long time intervals τ for these rounds may be in error by ±50 percent.

The variation of time interval τ with powder charge $\bar{m}$ is shown in figure 3. Integrating (36) from $t=-\tau$ to $t=0$ gives

38a) $m'\dot{y}(o) = \frac{\tau}{2} gD(-o) = 1.67\tau(10)^6$

38b) $m'\left[y(o)-\ell-h\right] = \frac{\tau^2}{6} gD(-o) = \frac{\tau}{3} m'\dot{y}(o)$

The greatest value of $\dot{y}(o)$ and $y(o)-\ell-h$ occur for rounds 4 and 7 for which $y(o)-\ell-h$ is negligible (.0006 ft.) as also is $\dot{y}(o)$ = 1.4 ft. per sec, but this would contribute a small amount .1 ft. to the shell displacement at ejection.

The conclusion is that the effect of velocity and displacement of the shell when recoil begins (t=o) are both negligible in every round, with the possible exception of Rounds (4) and (7) as to displacement at ejection.

The value of F_s together with inspection of the powder pressure curve show that even if the starting force F_s were zero, the shells inertia would not allow it to move sufficiently to alter the shape of the pressure curve. Hence a pre-rifled steel rotating band would have practically no effect on the rate of rise of the pressure.

VII. The Motion of the Shell and Its Numerial Test at Ejection.

The only other forces not yet considered are D_r, the action of the hydraulic brakes or recoil cylinder, and the variable part KY of the force of recoil springs.

Hence integrating (16) gives

$$39)\quad \dot{y}(t) = \frac{1}{m'}\left\{ M'\dot{Y}(t) + g\int_0^t D_r(t_1)dt_1 + tgD(+o) + \frac{gK}{3}\, t_e\, Y(t_e)\left(\frac{t}{t_e}\right)^3 \right\}$$

Integrating this gives

$$40)\quad y(t) - \ell - h = \frac{1}{m'}\left\{ M'Y(t) \cancel{+ g\int D_r(t_1)dt_1} + \frac{t^2 gD(+o)}{2} + \frac{gKt_e^2 Y(t_e)}{12}\left(\frac{t}{t_e}\right)^4 + g\int_0^t dt_2 \int_0^{t_2} D_r(t_1)\,dt_1 \right\}$$

where the variable part KY(t) of the spring's force has been represented by taking $Y(t) = Y(t_e)\left(\frac{t}{t_e}\right)^2$. Thus term turns out to be negligible.

The principle term in the second members of these equations is the first with factor M'. The term with factor D(+o) contributes to ejection velocities and displacements about 2 percent for service rounds and 7 percent for the two low pressure rounds. The factor g = 32.2 reduces pounds to poundals and $gD(+o) = 2.35(10)^6$ poundals.

The force $gD_r(t) = gSp_r(t)$ where $S = 335.6$ sq. inches is the area of the piston and $p_r(t)$ the liquid pressure whose time record was obtained. The time integrals were in most cases found by graphical integration. This is the principal part of the external force after it takes hold, but it does not begin to act until a time t_o = about .013 seconds, when it rises sharply and soon attains its maximum value. A sufficient representation is given by a linear relation from t_o to $t_o + \Delta$ followed by a constant value (maximum pressure p to ejection). If D_m denotes this final value, this may be written

41a)
$$\begin{aligned} D_r(t) &= o \text{ ---------------------- when } && t \leqslant t_o \\ &= \frac{D_m}{\Delta}(t - t_o) \text{ ------------ when } && t_o \leqslant t \leqslant t_o + \Delta \\ &= D_m = \text{const. ------------ when } && t_o + \Delta \leqslant t \end{aligned}$$

hence

41b)
$$\begin{aligned} \int_0^t D_r(t_1)dt_1 &= o \text{ --------------- when } && t \leqslant t_o \\ &= \frac{D_m}{\Delta}\frac{(t - t_o)^2}{2} \text{ ---- when } && t_o \leqslant t \leqslant t_o + \Delta \\ & D_m\left(t - t_o - \frac{\Delta}{2}\right) \text{--- when } && t_o + \Delta \leqslant t \end{aligned}$$

and

41c)
$$\begin{aligned} \int_0^t dt \int_0^{t_2} D_r(t,)dt_1 &= \text{ ------------ when } && t \leqslant t_o \\ &= \frac{D_m}{\Delta}\frac{(t-t_o)^3}{6} \text{-- when } && t_o \leqslant t \leqslant t_o + \Delta \\ &= \frac{D_m}{2}\left[(t-t_o-\Delta)(t-t_o) + \frac{\Delta^2}{3}\right] \text{ when } && t_o + \Delta \leqslant t \end{aligned}$$

The maximum value gD_m was about $23(10)^6$ poundals for service rounds. When D_m, t_o and Δ were estimated from the curves of $p_r(t)$ against time, the integrals at ejection when computed by (41b) and (41c) differed by not more than 6 percent of themselves from the results of graphical integration, except for the two low pressure rounds. Since this is the most important force, the graphical results were used. The results of the two methods are shown in table 3.

Table 3

Round	t_Δ sec	Δ sec	gD_Δ poundals	$g\int_0^{t_e} D_r(t_1)dt_1$ By $(41)_b$	$g\int_0^{t_e} D_r(t_1)dt_1$ By Graph. Integ.	$g\int_0^{t_e}dt_2\int_0^{t_2}D_r(t_1)dt_1$ By $(41)_c$	$g\int_0^{t_e}dt_2\int_0^{t_2}D_r(t_1)dt_1$ By Graph. Integ.
1	.0130	.0082	$23.8(10)^6$	$.393(10)^6$	$.400(10)^6$	$3.31(10)^3$	$\left\{\begin{matrix}3.38\\3.45\end{matrix}\right\|(10)^3$
2	.0127	.0075	23.8	.396	$.396(10)^6$	3.35	3.54
3	.0100	.0093	22.9	.413	---	3.80	3.95
4	.0100?	.0285?	4.5	.177	.174	3.62	3.40
5	.0140	.0080	12.9	.343	---	4.53	4.68
6	.0090	.0072	27.0	.461	---	4.02	4.04
7	.0100?	.0182?	6.0	.261	.217	5.67	4.64

For rounds (4) and (7) these are rough estimates. Only one pressure reading was obtained for round (4) and two for round (7).

Before proceeding with the assumption that D_0 is a constant, it is worth while to examine the forces more carefully. It was found to be a constant on the assumption that the frictional force exerted on the gun by its slide is $R = \mu Mg\cos\theta$. In the early part of recoil before the hydraulic brake D_r takes hold, this is obviously correct. The basis is found by estimating the position of the center of gravity of the gun. Rough estimation from drawings indicates that it is near the point of intersection of the axis of the gun and the axis of the trunion. The slide is a tube whose sections by a vertical plane through the axis of the gun are shown shaded in fig. 1. From drawings of the slide, it is found that in the initial position of the gun (before firing) the center of gravity of the gun is a distance $\bar{Y}$ + 4.3 ft. from the breech. Also, the lengths ℓ_1 and ℓ_2 of the sections of the slide are obtained from the drawings.

The action of the slide on the gun may be understood from the simplified conception indicated in fig. 1.

We at first assume the normal reactions on the gun are N_2 and N_1 at lower left and lower right end of the slide (the reaction on the upper section N_1 marked "later" is then absent). The sum of the force components normal to the gun's axis must be zero so that

42a) $$N_2 + N_1 = M \cos \theta \text{ (using pounds weight for unit of force)}$$

The moments of the forces about a horizontal axis through the center of gravity and perpendicular to the plane of the paper must also vanish. The expression for this is

42b) $$-\ell_2 N_1 + (\bar{Y} - \mu r_o)(N_2 + N_1) = r_1 D_r$$

Solving these two equations gives

42c) $$N_1 = \frac{\bar{Y} - \mu r_o}{\ell_2} M \cos \theta - \frac{r_1}{\ell_2} D_r$$

42d) $$N_2 = \left(1 - \frac{\bar{Y} - \mu r_o}{\ell_2}\right) M \cos \theta + \frac{r_1}{\ell_2} D_r$$

If the normal reactions N_1 and N_2 given by these equations are both positive the original assumption is justified and the friction is

$$R = \mu(N_1 + N_2) = \mu M \cos \theta \text{ pounds} = \mu Mg \cos \theta \text{ poundals}$$

as assumed in the expression for D_o. From the dimensions indicated in fig. 1 (taking $\mu = .047$) these become

$$N_1 = \frac{12.5 - .09}{15.1} M \cos \theta - \frac{3.17}{15.1} D_r = .82 M \cos \theta - .21 D_r$$

$$N_2 = \qquad = .18 M \cos \theta + .21 D_r > 0$$

Reference to table 3 shows that for service rounds the maximum value of D_r is $D_m = 24(10)^6$ poundals $= .75(10)^6$ pounds so that since $M \cos 8^\circ = .198(10)^6$ pounds

$$N_1 = (.82 \times .198 - .21 \times .75)\, 10^5 \text{pounds}$$

$$= (.162 - .158)\, 10^6 \text{ pounds which is positive.}$$

Since $\bar{Y}$ only decreases from 12.5' to 12.2' from start of recoil to ejection, it is probable that slide, trunion, and brake arm have been so designed that the hydraulic

We at first assume the normal reactions on the gun are N_2 and N_1 at lower left and lower right end of the slide (the reaction on the upper section N_1 marked "later" is then absent). The sum of the force components normal to the gun's axis must be zero so that

a) $N_2 + N_1 = M \cos \theta$ (using pounds weight for unit of force)

The moments of the forces about a horizontal axis through the center of gravity and perpendicular to the plane of the paper must also vanish. The expression for this is

b) $-\ell_2 N_1 + (\bar{Y} - \mu r_o)(N_2 + N_1) = r_1 D_r$

Solving these two equations gives

c) $N_1 = \dfrac{\bar{Y} - \mu r_o}{\ell_2} M \cos \theta - \dfrac{r_1}{\ell_2} D_r$

d) $N_2 = \left(1 - \dfrac{\bar{Y} - \mu r_o}{\ell_2}\right) M \cos \theta + \dfrac{r_1}{\ell_2} D_r$

If the normal reactions N_1 and N_2 given by these equations are both positive the original assumption is justified and the friction is

$$R = \mu (N_1 + N_2) = \mu M \cos \theta \text{ pounds} = \mu Mg \cos \theta \text{ poundals}$$

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$$N_1 = (.82 \times .198 - .21 \times .75)\ 10^5 \text{ pounds}$$

$$= (.162 - .158)\ 10^6 \text{ pounds which is positive.}$$

Since Y only decreases from 12.5' to 12.2' from start of recoil to ejection, it is probable that slide, trunion, and brake arm have been so designed that the hydraulic

brake does not throw the thrust N_1 on the upper right end of the slide before ejection, although this evidently may occur before full recoil is reached, but that does not concern the motion of the projectile. This opinion is strengthened by the fact that for ease of elevating at all angles the trunion axis must be very close to the center of gravity of gun and slide. This places the center of gravity of the gun somewhat further than the trunion axis from the breech so that $\bar{Y}$ is greater than the value used above, which makes it even more probable that N_1 remains positive throughout the interval up to ejection and therefore D_o is considered constant in this interval.

The numerical test of equation (40) at ejection time t_e is shown in table 4 where the shell displacement should be equal to the length of the rifling 49.8 ft. The values of M' given in (6d) are used here.

Table 4. Computed Travel of Shell at Ejection (M' = 211,200 lbs)

Round	$Y(t_e)$	t_e	$\frac{1}{12}gKt_e^2Y$	$M'Y(t_e)$	$g\int_0^{t_e}dt_2\int_0^{t_2}D_r dt_1$	$\frac{t_e^2 gD(t_o)}{2}$	Computed Travel	Error
1	.364 ft	.0336 sec.	.02(10)^3	76.88(10)^3	3.41(10)^3	1.33(10)^3	49.9 ft	+.1 ft
2	.362	.0331	.02 "	76.47	3.54	1.29	49.7	-.1
3	.357	.0327	.02 "	75.40	3.95	1.25	49.3	-.5
4	.303	.0635	.08 "	63.93	3.40	4.73	47.8	-2.0
5	.335	.0446	.04 "	70.72	4.68	2.34	49.3	-.5
6	.365	.0297	.02 "	77.09	4.04	1.04	49.8	.0
7	.321	.0622	.08 "	67.73	4.64	4.55	50.7	+.9
						Mean	49.6 ft	-.3 ft

-30-

In all but the two low pressure rounds the error is about what is to be expected in determining recoil $Y(t_e)$ at ejection. The uncertainty of the double time integral may account for the deviations of rounds (4) and (7). As a whole, this test of equation (40) may be considered satisfactory, the mean deviation from theory being about one-half of one percent.

If we now compare calculated and measured ejection velocities it must be remembered that the error in computing these by (39) will be about proportional to the error in determining recoil velocity $\dot{Y}(t_e)$ at ejection and since this is obtained by graphical differentiation of the recoil curves, this error may be expected to be several percent. The results are shown in table 5, the observed relative ejection velocities being the mean of these measurements with **three** pairs of contact fingers. The term with factor K is negligible.

Table 5. Calculated and Observed Ejection Velocities (relative to gun) (M' = 211,200 lbs)

Round	$\dot{Y}(t_e)$	$M'\dot{Y}(t_e)$	$g\int_0^{t_e} D_r dt$	$t_e g D(t_o)$	$\dot{y}(t_e)$	$\dot{y}$ observed	
	ft/sec	lb ft/sec			Computd.		
1	20.7	$4.372(10)^6$	$.400(10)^6$	$.079(10)^6$	2965 +160	2805	2793 2802 2816
2	20.5	4.330 "	.396 "	.078 "	2935 +150	2785	2760 2804 2788
3	20.5	4.330 "	.413 "	.077 "	2945 +140	2805	2782 2773 2861
4	8.2	1.730 "	.177 "	.149 "	1360 −110	1470	1439 1503 1463
5	12.7	2.681	.343 "	.105 "	1985 −175	2160	2145 2185 2152
6	23.0	4.858	.461 "	.070 "	3260 +90	3170	3690 lost 2651
7	9.2	1.941	.217 "	.146 "	1520 −10	1530	1518 1541 1530

The observed velocity for each round which was the average of three determinations by three pairs of fingers was considered accurate to about 1 percent as the three pairs were supposed to average out the effect of wobbling of the shell at the muzzle. This of course is not accurate for Round 6.

The excess of calculated over observed mean velocity expressed as a percent of the latter is shown in Table 6 in which y_c is calculated and y_o observed.

Table 6. Percent error in relative velocity at ejection (M' = 211200 lbs)

Round	1	2	3	4	5	6	7
$\frac{y_c' - y_o}{y_o}$	+5.7%	+5.4%	+5.%	-7.5%	-8.%	+3%	-.6%

[redacted]

It doesnot seem unreasonable to attribute a large part of these discrepancies to the error made in determining recoil velocity $\dot{Y}$ by graphical differentiation, especially in view of the good agreement with theory for the travel of the shell.

There is evidence of another kind which points in the same direction. The preceeding tables indicate that for the three service rounds 1, 2 and 3, the recoil velocities $\dot{Y}$ obtained by graphical differentiation are too large. When the recoil velocity curves are differentiated, the accelerations $\ddot{Y}$ will then be too high. This is shown when we attempt to plot the friction F between gun and shell by (26b) for these rounds. For certain values of the time in the neighborhood of the maximum acceleration of recoil the equation (26b) leads to negative values of the frictional force F. This absurdity may be caused in part or entirely by the pressure gage reading too low near the maximum powder pressure.

At this point, we must consider another possibility which arises from the uncertainty of the mass of the gun.

In the original notes, there are two places in which the total mass of the recoiling parts is totalled as in (6c) leading to the effective mass here used (6d).

In two other places the masses are not totalled but are itemized as follows:

Gun + breech =	180,320 lbs.
Yoke =	12,145
Recoil cylinder rode, pistons, etc.	2,242
giving M_o	194,707
Adding half springs $\frac{m_s}{2}$ =	3,558
$M = M_o + \frac{m_s}{2}$ = of the unloaded gun	198,265 lbs = effective mass
Adding $m + m + \bar{m}$ =	1,884 for service rounds

gives

43) $M' = M + m + \overline{m} = 200{,}150$ = effective mass of the loaded gun. This is just 5.5% lower than the mass used above and in the 1923 report. Its use would make the mean error in travel of the shell 2.8 feet low, but would make the ejection velocities about right for service rounds, but gives -13% error in velocities for rounds 4 and 5.

This mass corresponds closely to (1942) data for gun modification 5 instead of Mod. 3 furnished by the Navy Dept. Their data lead to

M' = 201,430 lbs. for service rounds.

The results of using (43) for the effective mass are shown in Table 7.

Table 7. Shell travel and ejection velocity (using M'=200,150 lbs)

Round	Travel $Y(t_e)$	Travel error	$\dot{y}(t_e)$ calc.	$\dot{y}(t_e)$ obs.	$\dot{y}_c - \dot{y}_o$	$\frac{\dot{y}_c - \dot{y}_o}{\dot{y}_o}$
			ft/sec.			
1	47.3'	-2.5'	2818	2805	+13.	.5%
2	47.1	-2.7	2792	2785	+ 7.	.2%
3	46.7	-3.1	2801	2805	- 4.	-.1%
4	45.4	-4.4	1298	1470	-172.	-12. %
5	46.8	-3.0	1855	2160	-305.	-14. %
6	47.2	-2.6	3100	3170	-70.	-2. %
7	48.3	-1.5	1448	1530	-82	-5. %
Mean	47.0	-2.8'				-3.2%

The velocities for the three service rounds are brought into excellent agreement with observation by this choice of M' but the other rounds are much worse than with the first choice of M'. This is a question of fact which ought to be resolved by getting information from the proper sources as to whether Modification 3 had 11,000 pounds more mass than Modification 5.

However, no choice of M' will make shell travel and ejection velocities both agree with observations. This also shows that the difficulties cannot be removed by a different theory of the inertial effect of the powder, since

changing m' has practically the same effect on the computed quantities as does a change of M'. If the calculated travel and velocity for a given round were both too small by the same percent (or too large) this could be accounted for, as explained above, by assuming the initial center of gravity of the powder to have been closer to (or further from) the breech than in case of uniform density as in equation (17e). Reference to tables 4 and 5 shows that round 4 is the only one which could be harmonized in this manner. A six percent decrease in m' would give a tolerable error in shell travel +.9 ft. with ejection velocity 1440 as compared with observed velocity 1470.

Although the comparison so far lends most weight to the larger M', there is further evidence to be considered which throws the balance in the other direction, unless the pressure gages are seriously in error. This is connected with the computation of the frictional force F between shell and gun. Before considering this, it is worth while to get an independent kind of check on the errors involved in graphical differentiation of recoil and again of recoil velocity.

VIII. Empirical Formula for Recoil and Test of Graphical Differentiation for Round 2.

In the original large scale drawings which exhibit recoil, its velocity and acceleration, there is one (Fig. 18 of the report quoted) in which two curves for velocity and two for acceleration are given, evidently obtained by different persons or at different times. They indicate that variations in velocities of 5 or 6 percent may be obtained. The errors in velocity are greatly magnified when these velocity curves are again differentiated to obtain acceleration of the gun. The second derivatives apparently differ from the smooth curve for acceleration by as much as thirty or forty percent in certain places.

To obtain an idea of the order of magnitude of errors, and an independent check, the empirical formula (7a) is here applied to the recoil curve for Round 2.

The graph of $\log_{10}Y$ as ordinate against $\log_{10}t$ is shown in two parts in fig. 4. The curve has a very high order of contact with its tangent whose slope is x = 2.63. With this value of α the constants A and B of equation (7a) are chosen to fit the observations at two points.

Table 8 shows that the computed and observed recoil never differ by .02" which is less than one-half of one percent near ejection. If. .02" is taken as a possible error, the equation (7a) then represents observations as closely as any other representation, and the velocity $\dot{Y}$ and acceleration $\ddot{Y}$ computed by (7b) and (7c) have as much validity as those obtained by graphical integration.

$\alpha = 2.63$ $\quad A = 48,800$ $\quad B = 3,440.$

Table 8. Recoil, velocity and acceleration by equation (7a,b,c) - Round 2

	t sec	Y (by 7a)	Y observ.	$\dot{Y}$ (by 7b)	$\dot{Y}$ graph-ical	$\ddot{Y}$ (by 7c) in/sec²	$\ddot{Y}$ (graph) in/sec²	$\log_{10} t^{\alpha}$
	.0025	in.		in/sec	in/sec	4,800		
1	.0050	.043	.04 in.	22.8	19	7,600	6,800	-6.0518
2	.0075	.125	.12	43.3	39.0	9,144	9,500	-5.5888
3	.0100	.263	.25	67.9	66.0	10,407	11,800	-5.26000
4	.0125	.466	.45	94.9	96.5	11,062	13,300	-5.00513
5	.0150	.738	.72	122.7	128.5	11,094	11,900	-4.79688
6	.0175	1.079	1.08	149.7	156.0	10,512	9,700	-4.62080
7	.0200	1.486	1.49	175.0	177.5	9,439	8,000	-4.46829
8	.0225	1.950	1.96	196.6	194.0	7,915	6,300	-4.33377
9	.0250	2.467	2.47	214.5	209.5	6,142	5,450	-4.21342
10	.0275	3.020	3.01	227.5	223.5	4,222	4,500	-4.10456
11	.0300	3.599	3.58	235.4	236.0	2,317	3,600	-4.00517
12	.0315	3.954	3.95	238.2	241.5	1,246	3,200	-3.94944
13	.0331	4.337	4.345	239.3	246.0	159	2,700	-3.89285

The velocities obtained by the two methods differ by 4 percent, the accelerations at maximum acceleration by 11 percent. The results are plotted in fig. 5, the dotted curves being graphical derivatives taken from the original curves drawn in 1923.

The curve representing Y(t) was drawn through all the points computed by equation (7a). The values of Y taken from the original curves were then indicated by crosses. Their closeness of fit is so remarkable as to suggest that there may be some

theoretical basis for the empirical equation (7a). No integral value of α would serve as well. It is safe to conclude that the new curves for velocity and acceleration are more accurate than the old ones, since they are not obtained by graphical differentiation. They confirm the suspicion stated above that the earlier values of relative velocity of the gun at ejection were too high. The new value of $\dot{Y}$ for round 2 gives a calculated relative ejection velocity of 2864 ft. per second which is 2 1/2 percent higher than the mean finger determination 2785. This is on the basis of $M' = 211,200$ for which tables 4 and 5 are constructed where the corresponding shell travel is 49.7 ft. (length of rifling 49.8 ft.). This particular round therefore favors the original choice of the heavier mass for the gun. The second alternative for M' gives ejection velocity 2730 which is 2 percent lower than observed while the shell travel is 2.7 ft. too small.

IX. Progress of the Projectile through the Barrel for Round 2

The progress of the shell or $y(t) - \ell - h$ has been computed by equation (40) noting that the last term which represents the effect of the variable part of the springs force is negligible. The double time integral of the force D_r of the recoil cylinder is evaluated by graphical integration. Two cases are considered.

Case (1) $M' = 211,200$ and case (2) $M' = 200,150$ lbs. The results are shown in Table 9 for Round 2, the values of Y in the third column of Table 8 being reduced to feet.

Table 9. Shell Travel, Round 2

	t sec	$g\frac{t^2}{2}D(to)$	$g\int_0^t dt_2 \int_0^{t_2} D_r(t_1)dt_1$	Case 1 M'Y	Case 2 M'Y	Case 1 $y - \ell - h$	Case 2 $y - \ell - h$
1	.0050	.03	---------	$.70(10)^3$	$.67(10)^3$	.45 ft	.43 ft
2	.0075	.07		2.11	2.00	1.33	1.26
3	.0100	.12		4.40	4.17	2.76	2.62
4	.0125	.19		7.92	7.50	4.96	4.70
5	.0150	.27	.05	12.65	12.00	7.92	7.53
6	.0175	.35	.11	19.01	18.03	11.90	11.30
7	.0200	.47	.32	26.22	24.85	16.51	15.67
8	.0225	.60	.57	34.50	32.69	21.80	20.70
9	.0250	.74	1.08	43.46	41.20	27.68	26.30
10	.0275	.89	1.67	52.98	50.20	33.95	32.25
11	.0300	1.06	2.38	63.01	59.71	40.62	38.60
12	.0315	1.17	2.88	69.52	65.88	44.96	42.75
13	.0331	$1.29(10)^3$	$3.54(10)^3$	$76.47(10)^3$	$72.47(10)^3$	49.70	47.25

$\ell + h = 7.862$ ft.

The curves for shell travel are shown for the two cases in fig. 6. The small circles represent the expansometer data. It is evident that the original choice of M' fits the expansometer data much better than the other choice. For this reason $\dot{y}$ and $\ddot{y}$ are computed for round 2 using M' = 211,200 and obtaining $\dot{Y}$ and $\ddot{Y}$ from (7b) and (7c). The results are shown in Table 10 and plotted in Fig. 6.

Table 10. Relative Velocity and Acceleration of Shell, Round 2

	t sec	$tgD(+o)$	$g\int_0^t D_r dt_1$	$M'\dot{Y}$	$\dot{y}(t)$	$gD(+o)$	$gD_r(t)$	$M'\ddot{Y}$	$\ddot{y}(t)$
					ft/sec				ft/sec^2
1	.0050	.012$(10)^6$	----	.40$(10)^6$	250	2.35$(10)^6$	---	134$(10)^6$	83.$(10)^3$
2	.0075	.017 "	----	.762 "	476	" "	1.0$(10)^6$	161. "	100x$(10)^3$
3	.0100	.02 "	----	1.195 "	740	"	1.6	183.	114. "
4	.0125	.025 "	----	1.67 "	1035	"	2.7	195.	122. "
5.	.0150	.030 "	.02$(10)^6$	2.16 "	1350	"	6.4	195.3	125. "
6.	.0175	.041 "	.05 "	2.63 "	1660	"	15.5	185.	124.
7	.0200	.047 "	.097	3.08 "	1970	"	20.8	166.5	116.
8	.0225	.052 "	.15	3.46 "	2240	"	22.1	139.	100 "
9	.0250	.059 "	.206	3.77 "	2460	"	22.7	108.	81
10	.0275	.065 "	.26	3.96 "	2620	"	23.0	74.	61.
11	.0300	.070 "	.32	4.14 "	2770	"	23.4	40.7	40.
12	.0315	.074 "	.36	4.20 "	2830	"	23.6	22.	29.3
13	.0331	.078 "	.396	4.22 "	2864	"	24.0	2.8	17.8

Table 10. Relative Velocity and Acceleration of Shell, Round 2

	t sec	$gD(\phi c)$	$g\int_0^t D_r dt_1$	$w\dot{r}$	$\dot{y}(t)$	$gD(\phi c)$	$gD_r(t)$	$w\dot{y}$	$\ddot{y}(t)$
1	.0050	.012(10)^6	----	.40(10)^6	250 ft/sec	2.35(10)^6	---	134(10)^6	63.4(10)^3 ft/sec^2
2	.0075	.017 "	----	.762 "	475	" "	1.0(10)^6	161. "	100.(10)^3
3	.0100	.02 "	----	1.195 "	740	"	1.6	133.	114. "
4	.0125	.025 "	----	1.67 "	1035	"	2.7	195.	122. "
5.	.0150	.030 "	.02(10)^6	2.16 "	1350	"	6.4	195.3	125. "
6.	.0175	.041 "	.05 "	2.53 "	1660	"	15.5	185.	124.
7	.0200	.047 "	.097	3.06 "	1970	"	20.3	160.5	116.
8	.0225	.052 "	.15	3.46 "	2240	"	22.1	135.	100 "
9	.0250	.059 "	.206	3.77 "	2460	"	22.7	108.	81
10	.0275	.065 "	.26	3.96 "	2620	"	23.0	74.	61.
11	.0300	.070 "	.32	4.14 "	2770	"	23.4	40.	40.
12	.0315	.074 "	.36	4.20 "	2830	"	23.6	22.	29.3
13	.0331	.078 "	.396	4.22 "	2864	"	24.0	3.6	17.3

X. Friction between Shell and Gun for Round 2

The external force is $D(t) = D(+o) + D_r(t) =$

$$\cong .073(10)^6 + D_r(t) \text{ lbs}$$

44) $$F(t) = \pi c^2 P(o,t) - \left[1 - .035\left(\frac{\ell+h}{y}\right)^2\right]\left[\frac{M'}{g}\ddot{Y}(t) + D(t)\right]$$

$$+ 58.6 \left[1 - .067\left(\frac{\ell+h}{y}\right)\right]\ddot{Y}(t) \quad \text{pounds weight}$$

Using the equation (7c) for $\ddot{Y}$ the steps for the computation of F by this formula are shown in Table 11 for the two choices of M' where $\ell + h = 7.862'$

Table 11. Friction on Shell, Round 2

	t sec	$\pi c^2 P(ot)$	D(t)	$\frac{M}{g}'\ddot{Y}$ Case 1	$\frac{M}{g}'\ddot{Y}$ Case 2	$\frac{\ell+h}{y}$ Case 1	$\frac{\ell+h}{y}$ Case 2
	0000	$.92(10)^6$	$.073(10)^6$	.0000	0000	1.00	1.00
	.0025	$2.19(10)^6$	$.073(10)^6$	$2.62(10)^6$	$2.49(10)^6$	.975	.985
1	.0050	3.47	.073	4.15	3.95	.945	.955
2	.0075	4.70	.073	5.00	4.75	.855	.868
3	.0100	5.40	.123	5.70	5.40	.740	.750
4	.0125	5.70	.160	6.05	5.73	.615	.627
5	.0150	5.83	.271	6.11	5.75	.500	.510
6	.0175	5.85	.543	6.10	5.73	.398	.410
7	.0200	5.70	.720	5.16	4.90	.323	.335
8	.0225	5.55	.760	4.32	4.10	.265	.275
9	.0250	4.32	.778	3.35	3.18	.220	.230
10	.0275	3.39	.789	2.31	2.19	.188	.196
11	.0300	3.03	.803	1.27	1.20	.162	.169
12	.0315	2.43	.808	.68	.65	.149	.155
13	.0331	2.23	.818	.087	.083	.136	.142

Table 11. Friction on Shell, Round 5

	t sec	$mc^2F(\omega t)$	D(t)	Case 1 $\frac{m'Y}{\bar{m}}$	Case 2 $\frac{m'Y}{\bar{m}}$	Case 1 $\frac{n+\lambda}{\lambda}$	Case 1 $\frac{n-\lambda}{\lambda}$
	.000	$.92(10)^6$	$.073(10)^6$	.0000	0000	1.00	1.00
	.0025	$2.19(10)^6$	$.073(10)^6$	$2.62(10)^6$	$2.49(10)^6$	.975	.985
1	.0050	3.47	.073	4.35	3.95	.945	.955
2	.0075	4.70	.073	5.00	4.75	.855	.868
3	.0100	5.40	.123	5.70	5.40	.740	.750
4	.0125	5.70	.160	6.05	5.73	.615	.627
5	.0150	5.83	.271	6.11	5.75	.500	.510
6	.0175	5.85	.543	6.10	5.73	.398	.410
7	.0200	5.70	.720	5.16	4.90	.343	.335
8	.0225	5.55	.760	4.32	4.10	.265	.275
9	.0250	4.32	.776	3.35	3.18	.220	.230
10	.0275	3.39	.789	2.31	2.19	.188	.196
11	.0300	3.03	.803	1.27	1.20	.162	.169
12	.0315	2.43	.808	.68	.65	.149	.155
13	.0331	2.23	.818	.087	.083	.136	.142

The pressure readings were rough and none were obtained for the highest pressures, the values here used being interpolated. In both cases equation (44) leads to the absurdity of a negative friction. It was believed that the pressure readings were too low. The computations are here carried out in the two cases to find how much they need to be increased to give positive F.

Table 12. Friction F on Shell, Round 2.

	t sec	Case 1 F lbs	Case 2 F lbs
	0	+ .85$(10)^6$	+ .85$(10)^6$
	.0025	- .56	- .43
1	.0050	- .59	- .40
2	.0075	- .20	+ .04
3	.0100	- .45	+ .04
4	.0125	- .38	- .06
5	.0150	- .49	- .09
6	.0175	- .70	- .33
7	.0200	- .06	+ .14
8	.0225	+ .51	+ .73
9	.0250	+ .22	+ .39
10	.0275	+ .31	+ .43
11	.0300	+ .97	+1.03
12	.0315	+1.75	+1.79
13	.0331	+2.14	+2.14

The results are plotted in fig. 7. The maximum powder pressure in these computations was estimated as 38,000 lbs/in^2. In case one it should have been 5,000 lbs higher, and in case 2, 2700 lbs higher to make the computed friction always positive. It is evident that even when the powder pressure and acceleration of the gun are more accurately known, the determination of F by (44) will be almost impossible throughout the greater part of the range. The fluctuations in the curve of fig. 7 may represent nothing more than accidental combinations of the errors in pressure and acceleration. The curve may be somewhat reliable near ejection. If so, and if the force is of the form (28) the rise near ejection must be attributed to F_e, that is, the choking effect of the thicker gun barrel at the muzzle. The term $\dot{y}^k$ representing kinetic

friction would be rising very slowly since the last four points of time in fig. 7 correspond to velocities $\acute{y}$ that are very close together as shown by fig. 6. It is, however, hard to believe that the force on the shell at ejection is more than double that required to start it.

XI. General Conclusions.

The theory of fluid motion in section 3 which leads to derivation of powder pressure at any point in terms of the reading of the pressure gage cannot be the source of the principal discrepancies observed. It is in fact less important than was at first believed.

The uncertainty as to the mass of the gun by 11,000 lbs, adds to the difficulties. On the whole, the earlier choice of the larger mass fits the observed results best, since it gives sufficiently correct travel of the shell. By using the empirical formula for round 2, the maximum acceleration is reduced 11 percent from the earlier values and the ejection velocity is brought within 2.5 percent of observed values. The lower value of M' would then have about the same error in ejection velocity but an error of about 3 ft. in shell travel. The expansometer data also fits the earlier choice of M'. The only explanation offered for the absurd negative values of the friction between shell and gun is that the powder pressure gages fail to record the maximum pressure by five thousand pounds in thirty-eight which is about 11 percent too low.

The estimates of the time by which the shell gets the start of the gun is about as accurate as powder pressure readings. The starting force required to start the shell corresponds to above 6,500 lbs/in^2 pressure. These estimates involve only the powder pressure measurements and the external force, the latter being the more accurately known. Neither the mass of the gun nor its acceleration enter into this determination of starting force or time interval τ. The discussion in section 6 shows that this time τ is so small that no appreciable displacement or velocity is obtained by the shell by the time the gun begins to recoil.

It is concluded that the determination of the relatively small force F(t) between gun and shell cannot be determined in general by the equation (26b). An alternative method would require some outside method of determining the elastic force F_e of equation (28) and the kinetic friction.

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11. General Conclusions.

The theory of fluid motion in section 3 which leads to derivation of powder pressure at any point in terms of the reading of the pressure gage cannot be the source of the principal discrepancies observed. It is in fact less important than was at first believed.

The uncertainty as to the mass of the gun of 11,600 lbs. adds to the difficulties. On the whole, the earlier choice of the larger mass fits the observed results best, since it gives sufficiently correct travel of the shell. By using the empirical formula for round 2, the maximum acceleration is reduced 11 percent from the earlier values and the ejection velocity is brought within 2.5 percent of observed values. The lower value of k' would then have about the same error in ejection velocity but an error of about 3 ft. in shell travel. The equipmentometer data also give the earlier choice of k'. The only explanation offered for the absurd negative values of the friction between shell and gun is that the powder pressure gages fail to record the maximum pressure by five thousand pounds in thirty-eight which is about 13 percent too low.

The estimates of the time at which the shell gets the start of the gun is about as accurate as powder pressure readings. The starting force required to start the shell corresponds to above 6,500 lbs/in² pressure. These estimates involve only the powder pressure measurements and the external force, the latter being the more accurately known. Neither the mass of the gun nor its acceleration enter into this determination of starting force or time interval. The discussion in section 6 shows that this time* is so small that no appreciable displacement or velocity is obtained by the shell by the time the gun begins to recoil.

It is concluded that the determination of the relatively small force F(t) between gun and shell cannot be determined in general by the equation (26a). An alternative method would require some outside method of determining the elastic force F_e of equation (33) and the kinetic friction.

The numerical test for round 2 indicates that F drops in value after the engraving and perhaps rises again as the shell approaches the muzzle although the latter may be an illusion. The conclusion was reached that the starting force could be considerably reduced by use of a pre-engraved shell and that the inertia of the shell would prevent its moving far before the pressure reaches its maximum. This opening up of the powder space would have little influence in the reduction of pressure in view of the rapid rate of rise of pressure already established when the shell begins to move. Although great increase in precision of measuring pressure and acceleration of recoil are desirable, the determination of F by this method will still be very inaccurate. It was concluded that F must be mostly an elastic reaction plus a kinetic friction.

The accurate representation of recoil by an empirical formula gives an alternative to graphical differentiation.

THE MOTION OF A ROTATING PROJECTILE

A Preliminary Study

by Prof. H. C. Richards, Ph.D.
Professor of Mathematical Physics, University of Pennsylvania.

Projectile D-5

The purpose of this paper is to review the forces acting upon a rotating projectile in its flight and to estimate their effects. Such a study is a necessary preliminary to any attempt to determine the deviation of a projectile from its path due to its rotation. Moreover it will serve to bring out the directions in which further investigation is most desirable.

The first part of the paper discusses the effects produced continuously on the direction and orientation of the shell because of its rotation, while the second part treats of the more or less transient effects which are produced at or near the muzzle. The third part consists of a review of the information available concerning the rate at which the speed of rotation decreases during the flight. While the discussion of the effects of the various forces is of course applicable to projectiles of any size or form, it was thought best to concentrate attention upon a particular example. The 14 inch naval armor piercing shell was selected as a type. The constants of this shell were obtained or calculated from the official specifications of the shell and gun, and are given in Table 1. As no values of the moments of inertia were available, an estimate was made by assuming a simplified body approximating to the true shell. The calculation is given in an appendix.

Table I.

Constants of Naval A. P. 14 Inch Shell

Quantity	Symbol	Numerical Value
Weight	W	1400 lbs.
Caliber	d	14 in.
Length	ℓ	49.44 in.
Distance of center of gravity from base.		19.3 in.*
Radius of tip		7 calibers **
Form Factor	i	0.70
Moment of Inertia about axis of figure	C	7.5 slug-sq.ft.
Transverse moment of inertia about c.g.	A	45.0 "
Ratio of moments of inertia		6.0
Initial velocity	v_o.	2800 ft.per sec.
Pitch of rifling		25 calibers
Initial angular velocity	ω_o	603 radians per sec.

* This value is estimated from that of a similar 16 inch shell which is given as 22.1 in.

** On account of the bulge of the cap, the shape of the shell approximates to a 5 caliber ogive.

PART I. THE EFFECT OF ROTATION ON THE MOTION OF A PROJECTILE.

The forces influencing the motion of a projectile are due to the gravitational field and to the reactions of the surrounding air. The latter may conveniently be considered as of three parts; first, the resistance opposing the motion of the projectile, which acts in the plane containing the line of flight and the axis; secondly, the frictional forces due to the rotation of the projectile; and lastly, the pressure effects due to the air carried around by the projectile in its rotation. It will be convenient to consider these various forces separately.

Gravity

The effect of gravity need be considered very briefly here as its influence is not modified by the rotation of the projectile. Its value may be taken as constant in magnitude and direction except for very long ranges, where the necessary corrections are well understood. It may be regarded as equivalent to

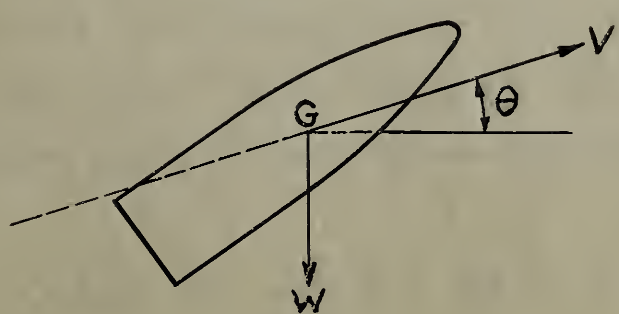

FIG. I.

(a) A force in the direction of the tangent to the path equal to $-W \sin \theta$ where θ is the angle made by the tangent above the horizontal plane (Fig. 1). This would produce a change of speed in the time δt equal to

$$(\delta v)_g = -g \sin \theta \, \delta t$$

(b) A force perpendicular to the path in a ~~horizontal (?)~~ vertical plane, equal to $W \cos \theta$. This would produce a downward change of direction of the tangent equal to

$$(\delta \theta)_g = -\frac{g}{v} \cos \theta \, \delta t$$

Air Resistance

The resistance of the air to the motion of a projectile is usually expressed by the formula

$$R = \frac{\pi d^2}{4} \cdot \rho \cdot i \cdot f(v)$$

where d is the caliber, ρ is the density of the air relative to that under standard conditions, and i is the "form factor", a quantity, assumed independent of velocity, depending upon the shape of the shell, and equal to unity for an arbitrarily selected standard shell (an ogive of two calibers). The factor $f(v)$ therefore represents the resistance of a standard shell of unit cross-section in air of standard density.

In passing, the question may be raised as to how far

the assumptions of this formula are correct. At the high velocities with which projectiles move, it is by no means certain that the resistance is proportional to the cross-section, nor that the form-factor is independent of the velocity. Direct experiments at high air velocities are desirable to test these points. However, the formula, with empirically determined form-factor, has proved satisfactory in the calculation of trajectories.(1)

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(1) Another uncertain factor in R is the air density ρ. An "altitude factor" is used to allow for the varying density with height but with the present long ranges and high elevations it is a question whether variations of temperature and pressure at high elevations may not introduce errors in range comparable with those produced by other sources. (See Greenhill, Engineering, Apr. 26, 1918). Variations of wind with altitude is another possible source of error.

The function of velocity $f(v)$ has been frequently discussed.(2) At low velocities it is proportional to the

---

(2) See the discussion in C. Cranz, Lehrbuch der Ballistik, Vol. I, p. 96. Also p. 57. This important work in four volumes will be referred to as Cranz. References to the first volume are to the second (1917) edition.

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square of the velocity but at about the velocity of sound it rises rapidly. The formula of Mayevski, which has been most commonly used in practical ballistics is

$$f(v) = A v^n$$

where A and n are constants which, however, take different

values for different ranges of velocities. One of the most recent discussions is by Mallock (P.R.S. 79, 1907, p. 262-273) who points out that for high velocities the function is nearly linear[3]. Mallock's formula is

[3] The linear law was first suggested by Chapel. See W. v. Scheve, Kriegstechn. Zeits. 10 (1907) p. 14, quoted by Cranz.

$$f(v) = 2.53\ (v - 850)$$

where the resistance is given in pounds per square foot and the velocity in feet per second; but the curve given by Mallock (l.c.) will above 1800 feet per second agree more closely with the formula

$$f(v) = 2.93\ (v - 1000)$$

and for these high velocities this latter formula also agrees better than Mallock's with the table given by Cranz (Vol. 1, page 58) from observations by the firm of Krupp. (See Fig.2). Thus when applied to the 14 inch shell of form-factor 0.7, the following results are obtained for the resistance R:

From Mayevski's formula	3821 lbs.
" Mallock's formula	3692 lbs.
" Mallock's curve	3935 lbs.
" Mallock's formula modified	3950 lbs.
" Krupp's table	3843 lbs.

From this it appears that here the most recent data are in substantial accordance and the formula given above may be used without serious error.

The effect of the air resistance on the translatory motion of a projectile has been frequently discussed, and

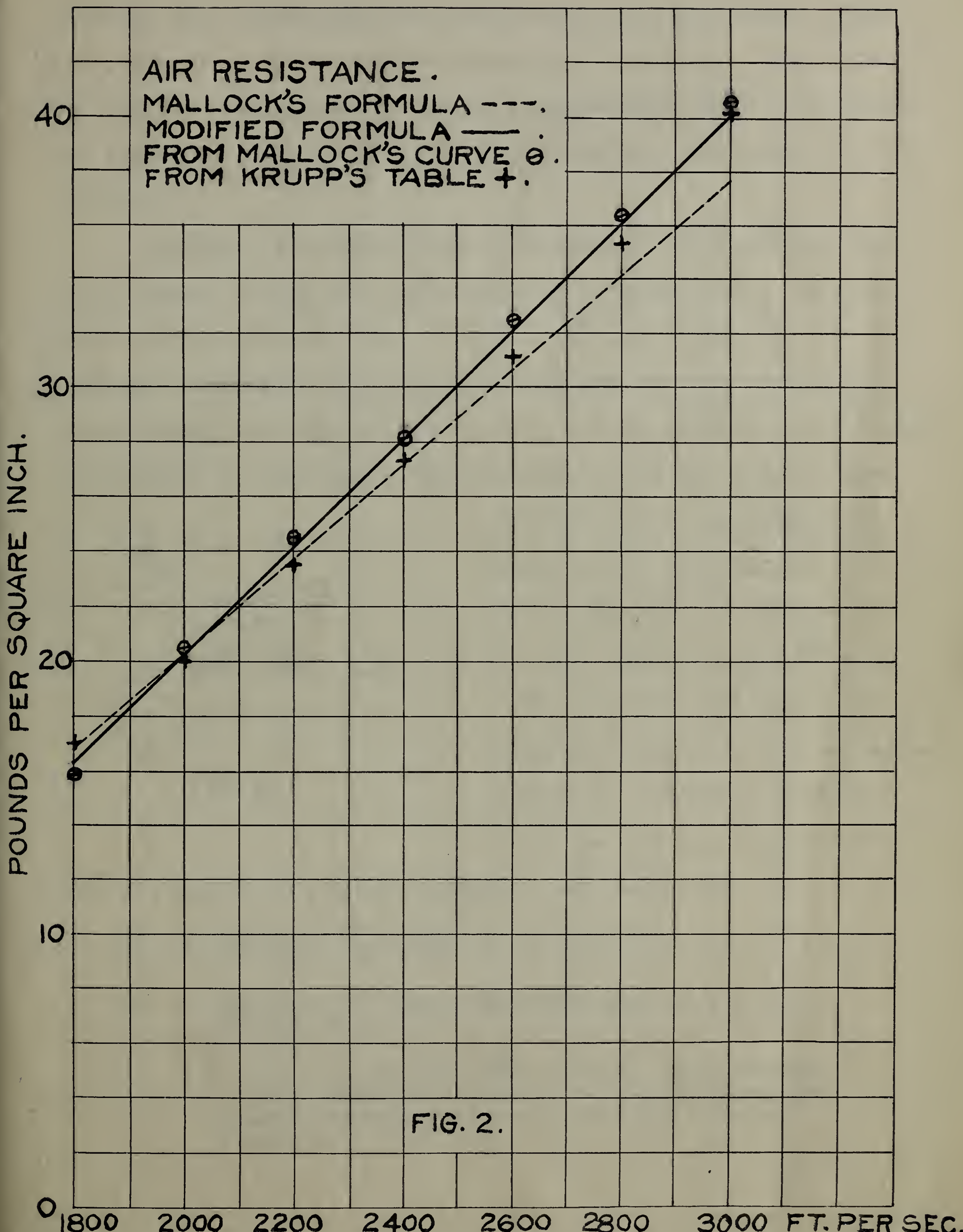
AIR RESISTANCE.
MALLOCK'S FORMULA ---.
MODIFIED FORMULA —— .
FROM MALLOCK'S CURVE ⊖.
FROM KRUPP'S TABLE +.
POUNDS PER SQUARE INCH.
40
30
20
10
0
1800
2000
2200
2400
2600
2800
3000
FT. PER SEC.

FIG. 2.

methods for determining the trajectory are well known. They are, however, aside from the present discussion. The data on air resistance were reviewed above because this also plays an important part in the effects on the projectile due to its rotation.

As long as a projectile is moving strictly end on, the only effect of the air resistance is to retard it. If however the axis is inclined to the direction of motion, the resultant resistance will also be inclined to this direction, and further its line of action will not in general pass through the center of gravity. Thus in Fig. 3, if the axis of the projectile is inclined by an angle α to the direction of motion V, the resistance of R' will make an angle β with $-V$, and will intersect the axis at a point P, the so-called center of pressure, distant a from the center of gravity G.

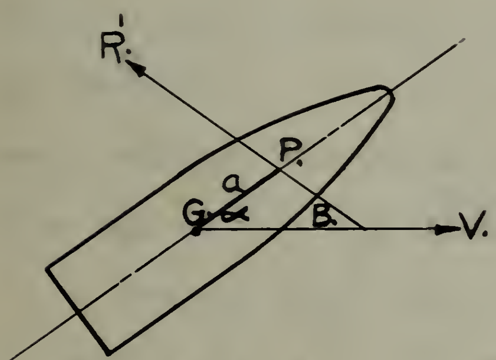

FIG. 3.

The air resistance R' is therefore equivalent to

(a) A component $R' \cos\beta$ in the direction of $-V$, acting at G.

(b) A component $R' \sin\beta$ perpendicular to V, also acting at G.

(c) A couple $R' a \sin(\alpha+\beta)$ about an axis through G perpendicular to the axial plane (i.e. the plane containing the axis and the line of flight).

These expressions involve the quantities R', a and β. R' is the resistance of the air to the tilted projectile. No experiments have apparently been made on the variation of resistance with tilt at high velocities. According to a formula given by Cranz (Vol. 1, p. 75), the increase is small and proportional to the $\sin^2$ of the angle; so that for small angles it may be taken practically equal to R.

The distance a of the center of gravity from the center of pressure is for similar shells proportional to the caliber. It may therefore be written

$$a = p\,d$$

where p is a numerical coefficient depending upon the design of the shell. It is also a function of the tilt but for small angles is usually assumed to be constant. It ~~is~~ however ~~not~~ ~~im~~probably ~~that it~~ also varies somewhat with the velocity.

The angle β also depends upon the form of the projectile. It varies with the tilt α and vanishes with it. For small angles it may be taken proportional to α, or

$$\beta = q\alpha ,$$

the numerical coefficient q being a function of the design of the shell and perhaps also of the velocity.

Unfortunately information as to the numerical values of the coefficients p and q is meagre. Röggla (M.A.v.G. 1912, p. 321) gives a table (from what source is not stated) which contains the values of q + 1(called k in his table) and ℓ'

the distance of the center of pressure from the point of the projectile, in calibers. Cranz (Vol. 1, p. 73) gives a theoretical discussion from which the same quantities may be deduced under an assumed law of variation of resistance with velocity for a projectile with a conical end corresponding in height to an ogive. Other data on the center of pressure may be found in Cranz but are not applicable to small angles of tilt. The values from Röggla's table and those calculated from Cranz are given in the following table: (See Fig. 4).

TABLE II

Radius of Ogive	Röggla		Cranz	
	$q + 1$	ℓ'/d	$q + 1$	ℓ'/d
0.5	1.	0.50	1	0.67
1.0	1.5	0.55	3	0.77
1.5	2.3	0.60	5	0.89
2.0	3.2	0.70	7	1.01
2.5	4.0	0.80	9	1.11
3.0	5.0	0.90	11	1.21
4.0	7.3	1.00	15	1.38
5.0	10. *	1.05*	19	1.53

* Extrapolated.

There is considerable discrepancy but it must be remembered that Cranz's values are calculated for a conical tip and therefore are probably higher than for the corresponding ogive. From the values of ℓ' the values of p may be calculated if the position of the center of gravity of the shell is known.

For the 14 inch shell which has been taken as the type, the center of gravity is 2.15 calibers from the point.

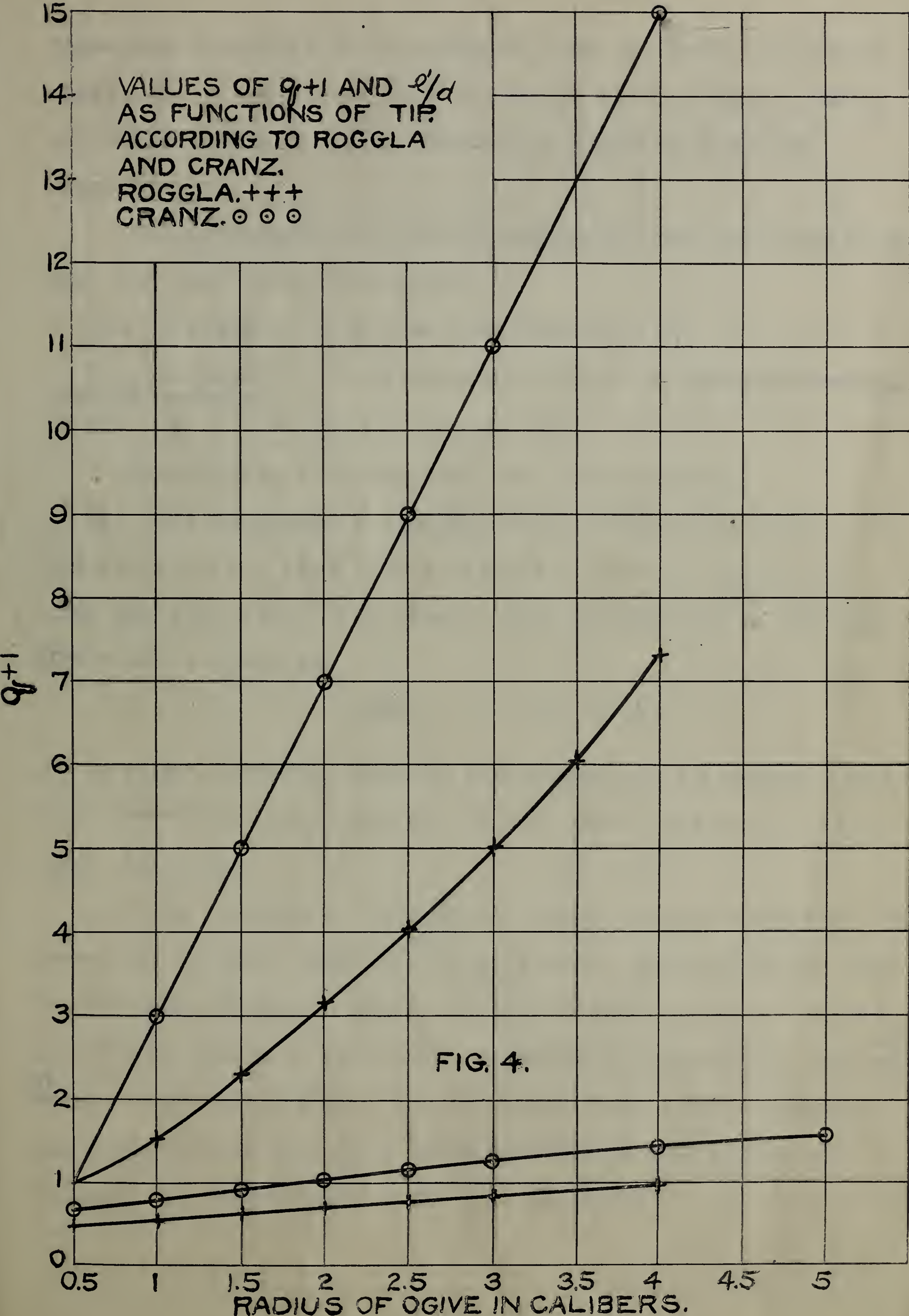
VALUES OF q+1 AND l'/d
AS FUNCTIONS OF TIP.
ACCORDING TO ROGGLA
AND CRANZ.
ROGGLA.+++
CRANZ.○○○
q+1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
RADIUS OF OGIVE IN CALIBERS.

FIG. 4.

Regarding the shell as an ogive of five calibers, we should obtain for p the value 1.10 by extrapolating Röggla's table or 0.62 from Cranz. The values of q would be 9 and 18 respectively.

The components of the air resistance may now be written, when the tilt is not too great,

(a) $R \cos\beta \doteq R$ in the direction of $-v_1$

(b) $R \sin\beta = q R \alpha$ perpendicular to v_1 ~~and the couple~~ and the couple

(c) $L_R = p(q+1) R d . \alpha$

The effects of these will now be considered.

(a) The component $R \cos\beta$ acts as a retarding force. It reduces to R for zero tilt and may be taken as equal to R when the tilt is not too great. The retardation in the time δt will be given by

$$(\delta v)_R = -\frac{R}{W} g \,\delta t$$

It is clear, however, that if the projectile is unstable and the tilt therefore large, the use of the normal value of R will lead to error.

(b) The component $R \sin\beta$, or $q R\alpha$, has no effect on the speed but affects the direction of motion, deflecting the path in the axial plane. Where the force acts as in the diagram, as will be the case with ordinary pointed projectiles, the direction of motion v will be deflected toward the instantaneous direction of the axis, thus diminishing the angle α. The rate of the deflection will be given by

$$(\delta\theta)_R = -\frac{R}{W}\frac{g}{v} \quad q\alpha\ \delta t$$

NOTE: Here and elsewhere the expression $(\delta\theta)$ does not necessarily mean the change in the angle θ (which is measured from the horizontal), but the shift of direction of the tangent line. Similarly $(\delta\alpha)$ will represent a shift of direction of the axis with an approximate subscript indicating the cause and $(\delta\psi)$ a change of the azimuth of the axial plane.

(c) The couple L_R because of the rotation of the projectile will produce or rather maintain a precession of the axis of the projectile about the line of flight. Thus if the resistance acts as in Fig. 5 , and the rotation of the shell is right handed, the point of the shell will be tilted upward from the plane of the paper, and the precession will be clockwise about the line of flight $\underset{\sim}{V}$, as viewed from the rear. The precession, i.e., rotation of the axial plane may be determined from the vector diagram. (Fig. 5)

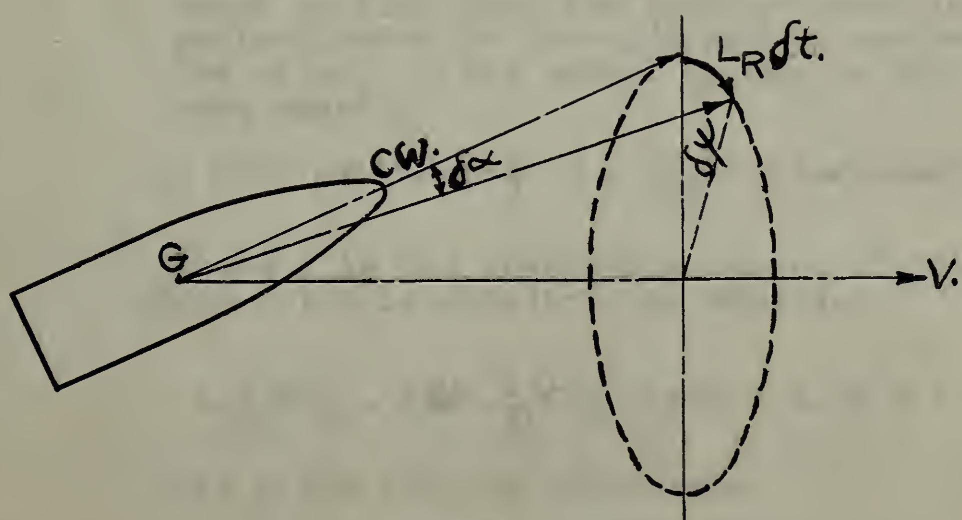

FIG. 5.

Here $L_R \delta t$ is the change in the time δt of the angular momentum $C\omega$, C being the axial moment of inertia and ω the angular velocity of spin. As seen from the diagram

$$L_R \delta t = C\omega(\delta\alpha)_R = C\omega \sin\alpha(\delta\psi)_R \doteq C\omega\alpha(\delta\psi)_R$$

Substituting for L_R its value $p(q + 1) R d . \alpha$ obtained above (page 12), the expression for the precession becomes

$$(\delta\psi)_R = \frac{p(q + 1) R d}{C\omega} \delta t$$

and the change in the direction of the axis

$$(\delta\alpha)_R = \frac{p(q + 1) R d}{C\omega} \alpha\delta t = \alpha(\delta\psi)_R$$

It will be noticed that the rate of precession does not depend on the angle of tilt, and that the change of the tilt is proportional to the tilt itself.

> NOTE: In the foregoing discussion the rate of rotation of the projectile has been assumed so high that the angular momentum perpendicular to the axis may be neglected. The effect of the moment L_R may be written more exactly
>
> $$L_R\, dt = C\omega \sin\alpha\, d\psi - A\frac{d\psi}{dt}\sin\alpha\cos\alpha\, d\psi$$
>
> (where A is the transverse moment of inertia) from which is obtained the equation
>
> $$A\left(\frac{d\psi}{dt}\right)_R^2 - C\omega\left(\frac{d\psi}{dt}\right)_R + p(q + 1) R d = 0$$
>
> This gives for the precession
>
> $$\left(\frac{d\psi}{dt}\right)_R = \frac{C\omega}{2A}\left[1 - \sqrt{1 - \frac{4p(q + 1) A R d}{C^2\omega^2}}\right]$$

or as an approximation

$$\left(\frac{d\psi}{dt}\right)_R = \frac{p\,(q+1)\,R\,d}{C\,\omega}\left(1 + \frac{p\,(q+1)\,A\,R\,d}{C^2\,\omega^2}\right)$$

The more exact solution also indicates the criterion of stability, the condition being

$$\frac{1}{\sigma} \equiv \frac{4\,p\,(q+1)\,A\,R\,d}{C^2\,\omega^2} < 1$$

The quantity σ here defined is taken as the measure of the stability of the shell.

In the case of the typical shell, taking as above $p = 1.10$ and $q = 9$ with $R = 3900$ lbs,

$$\frac{1}{\sigma} = \frac{1}{2.27}$$

so that the shell is initially stable; and since the resistance R decreases much more rapidly than the angular velocity it follows that it remains stable throughout its flight. Thus when fired with a range of 25,000 yards, the final velocity is 1373 ft./sec. from which the value of R may be estimated as 1040 lbs. As the time of flight is 42 seconds and the logarithmic decrement of the spin, as estimated later, is .0025 the final velocity of rotation will be 543. These data give the result

$$\frac{1}{\sigma} = \frac{1}{6.9}$$

which indicates a much greater stability than at the beginning of the flight. The actual values of the precession would be

Initial	12.6	radians	per	sec.
Final	3.4	"	"	"

or at the rate of about two cycles per second at the start and about half a cycle per second at the end of the flight.

Frictional Forces

A considerable part, and if the projectile is not rotating, the whole, of the frictional force of the air will be in the axial plane and will therefore be included in and inseparable from the resistance R. With a rotating projectile there will be additional tangential forces. These when the projectile is traveling end-on will be symmetrical and the sole effect will be to decrease the spin. When there is a tilt, the forces on the front and rear of the projectile will be unequal and unsymmetrical so that there will be a resultant force perpendicular to the axial plane. For the same reason the axis of the frictional couple will no longer be directed along the axis of the projectile. The effect may be examined by considering the diagram Fig. 6. Here the rotation of the projectile is supposed to be right handed as indicated by the vector M. The frictional forces may for simplicity be

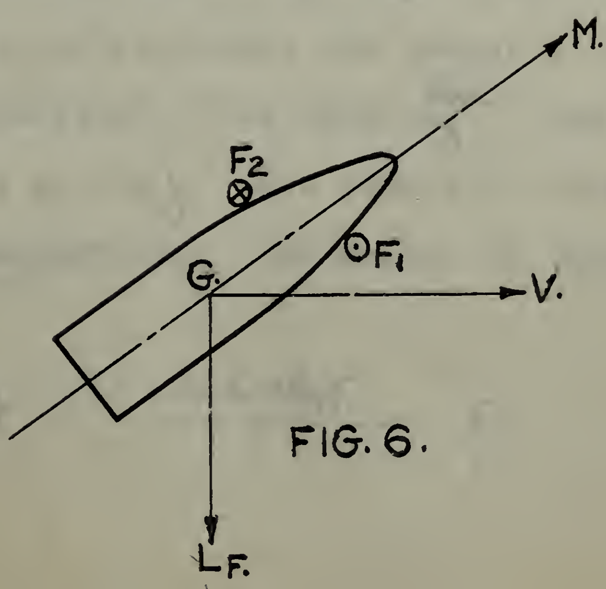

FIG. 6.

considered equivalent to two, each perpendicular to the axial plane, a force F_1 on the front surface of the projectile acting outward from the plane of the paper, and an inward force F_2 on the rear. These again are equivalent to

(a) A force $F = F_1 - F_2$ perpendicular to the axial plane acting at G, and a couple L_F whose axis is in the axial plane not in general coincident with the axis but making an angle γ with its negative direction. This may be resolved into two couples

(b) $L_f \cos \gamma$ in the direction of the negative axis.

(c) $L_f \sin \gamma$ at right angles to it.

Corresponding to this analysis, the frictional forces will produce three effects upon the projectile.

(a) The resultant F will deflect the path perpendicularly to the axial plane in the direction (F_1 being the greater force) given by turning a right handed screw from direction of v to that of M (i.e. in the direction of the vector product $V v M$). The amount of the deflection in the time δt is given by

$$(\delta \theta)_F = \frac{F}{W} \frac{g}{v} \delta t$$

$(\delta \theta)_F$ here does not represent the change in the angle θ but the change in direction of the tangent line. (See note page 13)

(b) The couple $L_F \cos \gamma$ acts simply to retard the rate of spin of the projectile. The amount of this decrease may be written

$$\delta \omega = - \frac{L_F \cos \gamma}{C} \delta t$$

(c) The couple $L_F \sin\gamma$ produces a precession causing the momentum axis to rotate in the axial plane toward the line of flight and so decrease the tilt. The amount of shift of the axis will be given by

$$(\delta\alpha)_F = -\frac{L_F \sin\gamma}{C\,\omega}\,\delta t$$

as is readily seen by the vector diagram. Fig. 7.

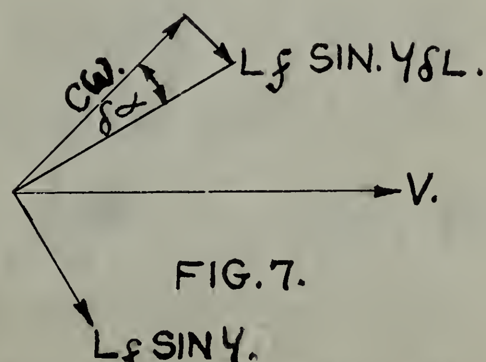

FIG.7.

Unfortunately no data are available as to the value of the resultant force F or the angle γ except that they approach zero with the tilt α, Scarcely any more information is at hand concerning the value of the couple L_F. The evidence on the law of decrease of spin of a projectile is reviewed in the third part of this paper, and it is there shown that the few known observations are not inconsistent with theory ~~is writing~~ which suggests the formula

$$L_F \cos\gamma = -k\,\frac{C}{d}\,\omega$$

Here k is a constant determined from the experimental results to be about .09 cm./sec. or .035 in./sec. This would give for the loss of spin the value

$$\delta\omega = -\frac{k}{d}\,\omega\,\delta t$$

(e) The couple $L_y \sin\gamma$ produces a precession causing the momentum axis to rotate in the axial plane toward the line of flight and so decrease its tilt. The amount of which of the axis will be given by

$$\delta\alpha_y = -\frac{L_y \sin\gamma}{C\,\omega}\,\delta t$$

as is readily seen by the vector diagram. Fig. 7.

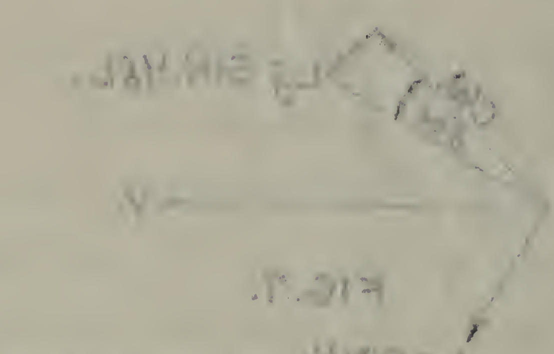

FIG. 7.

Unfortunately no data are available as to the value of the resultant force L or the angle γ except that they approach zero with the tilt α. Scarcely any more information is at hand concerning the value of the couple L_y. The evidence on the law of decrease of axis of a projectile is reviewed in the third part of this paper, and it is there shown that the few known observations are not inconsistent with theory ~~assuming~~ which suggests the formula

$$L_y \cos\gamma = -k\,\frac{C}{B}\,\omega$$

Here k is a constant determined from the experimental results to be about .09 cm./sec. or .035 in./sec. This would give for the loss of axis the value

$$d\alpha = -\frac{k}{B}\,\omega\,\delta t$$

and 45-47.

and for the spin at any instant

$$\omega = \omega_0 \, e^{-kt/d}$$

The experiments were all made with shells of from 3 to 4 1/2 inches caliber, and it is uncertain whether the formula may be extended much beyond these limits. If it is permissible to apply the results to the 14 inch shell, the value of the frictional couple at the beginning of the flight would be

$$L_F \cos\gamma = .035 \frac{7.5}{14} 603 = 11.3 \text{ lb-ft}$$

The loss of spin of this shell would in general become

$$\delta\omega = - \frac{.035}{14} \omega\delta t = .0025 \, \omega\delta t$$

and the speed of rotation at any time

$$\omega = 603 \, e^{-.0025 t}$$

These results perhaps give the order of magnitude of the frictional couple and its effect. They may also throw a little light on the other quantities involved. The other moment $L_F \sin\gamma$, at least for small angles of tilt, has a smaller value. Its effect, however, is continuous as it always acts to decrease the tilt. Thus a couple of 2 lbs-ft would diminish the tilt by about 1.5 minutes of arc per second.

To estimate the value of F we may take the extreme case that $F_2 = 0$, when

$$L_F \cos\gamma \doteq F_1 \frac{d}{2}$$

This shows that F can hardly be more than 20 pounds. A force of this magnitude would deflect the line of flight at the rate of about 0.5 minute of arc per second; but as its direction is constantly changing because of the rotation of the axial plane, the effect is negligible except perhaps in ~~exceptional~~ cases where the tilt of the projectile is preponderatingly in one direction.

Force due to Air Pressure

There remains to be considered the effect on the projectile of the air carried round by it in its rotation. This is sometimes known as the Magnus effect. Here again there will be no effect if the projectile is traveling end on. But if the axis is tilted there will be produced an inequality of pressure on the two sides of the axial plane.

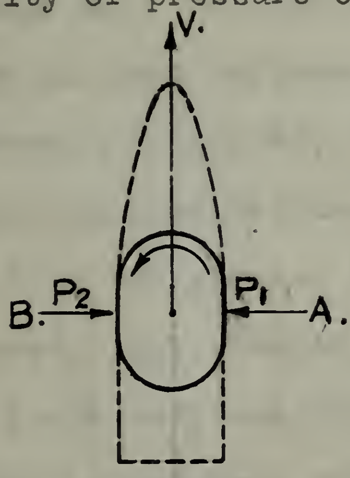

FIG. 8.

Consider a section parallel to the line of flight perpendicular to the axial plane. (Fig. 8). As drawn, the point of the projectile is above the plane of the paper and the direction of rotation in the sectional plane will be as shown. On the side A the rotating air is advancing and will be more condehsed than on the side B where it is (relatively) retreating. The pressure on the side A will consequently be greater than that on the side B. If P_1 and P_2 are the integral components of these pressures taken over the two sides of the

projectile there will be a resultant force

$$P = P_1 - P_2$$

perpendicular to the axial plane. (The components in the axial plane are included in the resistance R.) This force is in the opposite direction to the frictional force F. Furthermore, unless this resultant happens to act through the center of gravity of the projectile, there will be a couple L_p with its axis in the axial plane. This couple will be opposite to the frictional couple and will therefore tend to increase the tilt.

For spherical projectiles and low velocities the effect of this force exceeds that of the frictional force. This has been shown by observations on spherical shells, and is also familiar in the well known curving of a rotating base ball or golf ball. As the speed increases, the effect is less noticeable and it has been suggested by Henderson (P.R.S. 82, 1909, p. 555) that it becomes negligible at high velocities because the projectile is carried away from the air too fast for the lateral pressure to be produced. Experiments as to the amount and limits of this effect are very desirable. At present all that can be said is that in the case of high velocities such as those here considered the frictional force F will be replaced by F-P and the frictional moment affecting the tilt by $L_F \sin\gamma - L_p$, where the appended terms are small or negligible.

Summary

It will be seen that the forces which have been considered produce four different kinds of effect; they modify the magnitude and direction of the velocity, and the magnitude and direction of the spin.

(1) The magnitude of the velocity is modified by the component of gravity $W \sin \theta$ (increased or diminished according as θ is negative or positive) and is decreased by the component of the resistance $W \cos \beta$.

(2) The direction of motion θ is curved downward in a vertical plane by the component of gravity $W \cos \theta$, is bent towards the instantaneous position of the axis by the componentof the resistance $R \sin \beta$, and is deflected at right angles to the axial plane by the difference of the forces F and P.

(3) The magnitude of the spin is diminished by the component of the frictional couple $L_F \cos \gamma$.

(4) The direction of the axis of spin is deflected at right angles to the axial plane by the resistance couple L_R, and in this plane by the difference of the frictional and pressure couples $L_F \sin \gamma - L_p$, the net effect being to diminish the tilt.

The amounts of these various effects are summarized in the following table:

Table III

Force	Changes in v Magnitude	Changes in v Direction	Changes in ω Magnitude	Changes in ω Direction
Gravity	$- g \sin \theta \, \delta t$	$- \frac{g}{v} \cos \theta \, \delta t$ in vertical plane	-----	-----
Resistance	$- \frac{R}{W} \, g \, \delta t$	$\frac{R}{W} \frac{g}{v} q \alpha \delta t$ toward axis	------	$\frac{p(q+1)Rd}{C \omega} \, \alpha \, \delta t$ $\perp$ to axial plane
Friction	----------	$\frac{F}{W} \frac{g}{v} \quad \delta t$ $\perp$ to axial plane	$- \frac{k}{d} \omega \delta t$	$\frac{L_F \sin \gamma}{C \omega} \quad \delta t$ towards line of flight
Pressure	----------	$\frac{P}{W} \frac{g}{v} \quad \delta t$ opposite to F	-----	$\frac{L_p}{C \omega} \quad \delta t$ away from line of flight

The changes in direction may be represented graphically by the following device. Describe a large sphere about the center of gravity of the projectile and suppose it to have the motion of translation of the projectile but no rotation. Let the positive prolongation of the tangent to the trajectory and of the momentum axis intersect the sphere at points v and M respectively. The distance between v and M, measured in terms of the radius of the sphere, will be the tilt of the projectile α. As the projectile moves the points v and M will describe curves on the sphere which will indicate the shift in direction of these vectors. This is illustrated in Fig. 9. Let v and ~~N~~ M be the points represent-

ing the directions of the tangent and axis at any instant. The line ν M will then represent the tilt α . In a small interval δt the point ν would be shifted by gravity to the point g, by the air resistance to r, by friction to f and by the pressure to p. The resultant of these displacements will cause the point ν to shift to ν' the new position of the tangent. During the same interval the point M would be shifted to R by the resistance, to F by the friction and to P by the pressure. The resultant shift would bring it to M'. The new value of α would then be represented by ν'M'.

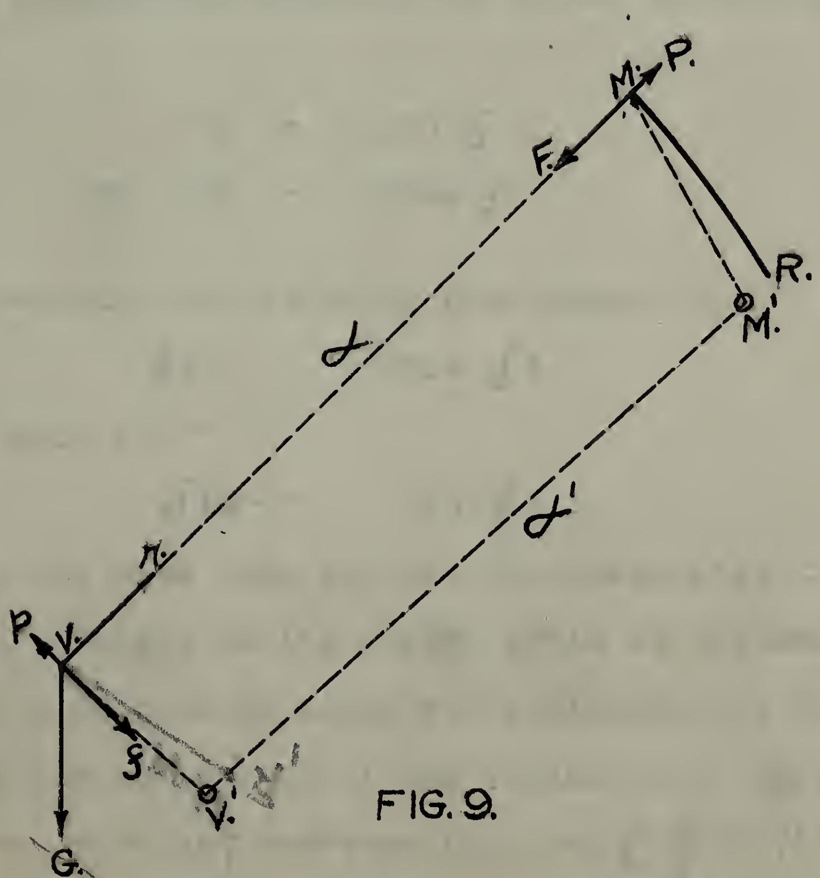

FIG. 9.

The relative values of the displacements, which are intentionally misrepresented in the diagram in order to bring out more clearly the directions, may be illustrated by a special case. For this purpose the values were computed ~~from~~ for the typical shell, using plausible data as suggested in the previous discussion. The velocity was taken as 2800 ft. per sec., at 15 degrees from the horizontal, and a tilt of 1 degree was assumed. The component displacements of the line of flight were

$$v_g = .0110\ \delta t$$
$$v_r = .0050\ \delta t$$
$$v_f - v_p = .00016\ \delta t$$

and the simultaneous variations in the direction of the axis.

$$MR = 0.221\ \delta t$$
$$MF - MP = .0044\ \delta t$$

At the sametime the velocity has changed by

$$\delta v = -\ 97.4\ \delta t$$

and the spin by

$$\delta \omega = 1.51\ \delta t$$

If the data were sufficiently accurate, the behavior of the projectile in its flight could be followed by drawing such a diagram to scale and continuing it from v' and M' using the new values of the variables. Such diagrams have been drawn by Henderson (P.R.S . [A] 82, (19-), p. 555); Röggla (M.A.G. Wien 43, 1912, p. 317); and Burzio (Rivista

di Artigliera & Genio 35, 1918, II p. 1).

These curves show the general characteristics of the motion. They illustrate for instance the fact that usually the axis is inclined (with right handed rifling) more to the right than the left, so that the resistance component R sin β deflects the projectile on the whole to the right producing the " drift". Since small variations in the constants would produce considerable changes in the form of the curve it was not thought desirable to trace such a curve. When more accurate information is available as to the values of the quantities involved, the construction of such curves may throw considerable light upon the motion of the projectile.

PART II. EFFECTS DUE TO DISTURBANCES AND TO WANT OF SYMMETRY.

In the previous discussion it has been assumed that the motion of the projectile has been affected only by the continuously acting forces due to gravity and to the air. It has also been assumed that the projectile is a symmetrical solid rotating about its axis of figure. In the present section will be considered the effect first of disturbances acting on the projectile, especially at or near the moment of its ejection from the gun, and secondly of a lack of symmetry. Under the first head may be included effects caused by

(1) Vibrations of the Gun.
(2) Jump and Whip of the Gun.
(3) Blast from a Neighboring Gun.
(4) Irregular Escape of the Powder Gases and Direct Action of the Blast.

While the second would include

(5) Lateral Displacement of the Center of Gravity.
(6) Angular Displacement of the Principal Axis of Inertia.

Since the perturbations produced by these causes are not symmetrical with respect to the line of flight they would in general produce a deflection of the path. It is desirable therefore to estimate the magnitude of these possible disturbances.

Before taking up these results separately, it will be convenient to consider briefly the theory and the effect (a) of an impulsive moment applied to the spinning projectile;

(b) of a deviation of the principal axis of inertia from the axis of figure. To simplify the discussion it will be assumed in each case that no other forces are acting upon the projectile.

(a) Effect of an Impulsive Moment

Let a symmetrical projectile of principal moments of inertia A and C be rotating about its axis of figure with angular velocity ω_o. Its angular momentum will then be

$$M_o = C\,\omega_o$$

Let now a couple of moment L act for a brief time about a perpendicular axis. The angular momentum produced about this axis will be

$$M' = \int L\,dt$$

so that the angular velocity produced will be

$$\omega' = \frac{M'}{A}$$

The resultant angular velocity and momentum ω and M are obtained by composition as in Fig. 10. The instantaneous axis

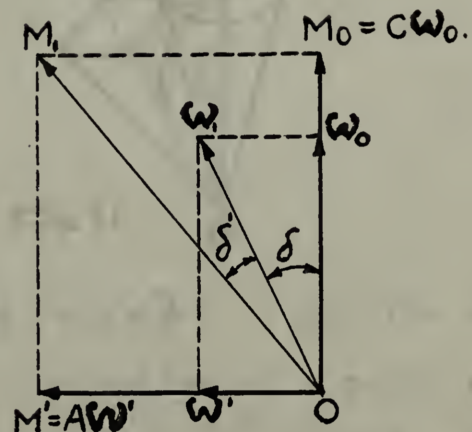

FIG. 10.

of rotation will make an angle δ with its original direction, where

$$\tan \delta = \frac{\omega'}{\omega_0}$$

while the axis of momentum will (if $A > C$, as is the case with elongated projectiles) be shifted through a greater angle $\delta + \delta'$ where

$$\tan (\delta + \delta') = \frac{A\ \omega'}{C\ \omega_0} = \frac{A}{C} \tan \delta$$

Since, as assumed, no other forces are acting, the angular momentum M will subsequently remain constant in magnitude and direction. The changes in the vectors may then be represented by considering the figure to rotate about the line OM. The directions of the axis of the projectile and of the instantaneous axis of rotation will describe cones of amplitudes $\delta + \delta'$, and δ respectively; and the motion of the projectile about its instantaneous axis ω may be represented by that of a cone of amplitude δ rolling upon a fixed cone of amplitude δ', (Fig. 11), the axis of the rolling cone corresponding to that of the projectile.

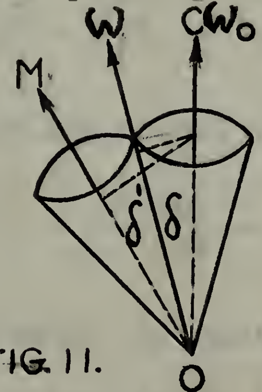

FIG. 11.

To obtain the rate Ω at which the axis OC will rotate about the line ~~ON~~ OM, the motion of a point on the axis at unit distance from O may be considered. The instantaneous velocity of this point is $\omega \sin \delta$ perpendicular to the plane through the axes, from which it follows that its angular motion about OM will be given by

$$\tan \delta = \frac{\omega'}{\omega}$$

while the axis of momentum will (if $C > A$, as is the case with elongated projectiles) be shifted through a greater angle $\delta + \delta'$ than δ

$$\tan(\delta + \delta') = \frac{A\omega'}{C\omega} = \frac{A}{C}\tan\delta$$

Since, as assumed, no other forces are acting, the angular momentum μ will necessarily remain constant in magnitude and direction. The changes in the vectors may thus be represented by considering the figure to rotate about the line OM. The directions of the axis of the projectile and of the instantaneous axis of rotation will describe cones of amplitudes $\delta + \delta'$ and δ' respectively; and the motion of the projectile about its instantaneous axis ω may be represented by that of a cone of amplitude δ rolling upon a fixed cone of amplitude δ', (Fig. 11), the axis of the rolling cone corresponding to that of the projectile.

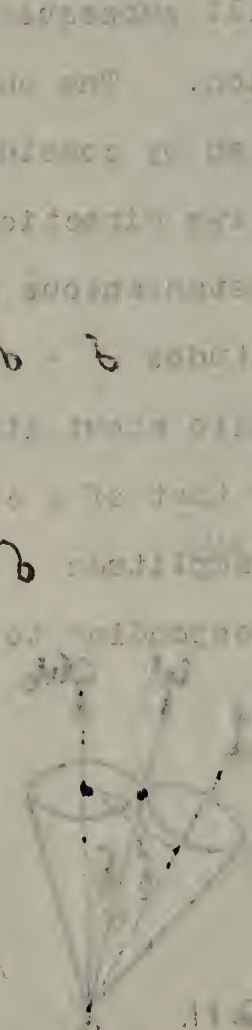

FIG. 11.

To obtain the rate Ω at which the axis OC will rotate about the line OM, the motion of a point on the axis at unit distance from O may be considered. The instantaneous velocity of this point is $\omega \sin\delta$ perpendicular to the plane through the axes, from which it follows that its angular motion about OM will be given by

$$\Omega = \frac{\omega \sin \delta}{\sin(\delta + \delta')} = \frac{\omega'}{\sin(\delta + \delta')}$$

and therefore by

$$\Omega = \frac{C\,\omega_0}{A \cos(\delta + \delta')}$$

The axis of projectile will therefore return to its original direction at intervals equal to

$$T = \frac{2\pi A \cos(\delta + \delta')}{C\,\omega_0}$$

The average angular deviation of the axis from its initial direction will be $\delta + \delta'$, and the maximum will be twice this amount. The direction of the average deviation will be that of the axis of the impulsive couple.

Applying these results to the case of the 14 inch shell where $A = 45$, $C = 7.5$ $\omega_0 = 603$, the average tilt, supposed small, in terms of the applied impulsive moment, is given by

$$\delta + \delta' = \frac{M}{4523}$$

or in terms of the angular velocity imposed

$$\delta + \delta' = \frac{6\,\omega'}{603} = .01\,\omega$$

Expressing the result in another way, to produce a displacement of average amplitude 1 degree would require an impulsive moment amounting to

$$M' = \frac{4523}{57.3} = 79 \text{ lb.-ft.-sec.}$$

or would require an imposed transverse angular velocity of

$$\omega' = \frac{100}{57.3} = 1.75 \text{ radians per second}$$

The period will be independent of the amplitude and will be

$$T = \frac{2\pi \times 6}{603} = .0626 \text{ second}$$

As has been stated, it is here assumed that the projectile is subjected to no forces except the impulsive force producing the disturbance. The results therefore are strictly applicable only to the brief interval after leaving the muzzle when the propulsive force of the blast is practically balanced by the resistance of the air.*

---

* The force of gravity of course has no effect on the rotation of the shell.

When the latter predominates, the motion will be modified by a precession around the line of flight. The tip of the projectile will no longer describe a circle but a looped curve similar to the diagram Fig. 12 (taken from Cranz, Vol.I, p.361), returning however as before periodically to the original direction. The amplitude of the displacement and the

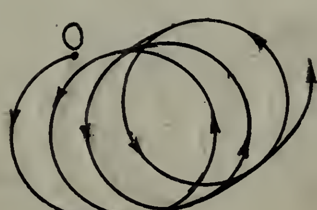

FIG.12.

interval between the successive passages through the original direction will each be somewhat increased.* However,

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* Approximately, when the shell is fairly stable, by the friction $1/2\sigma$, where σ is the " stability". (See above, p. 15).

---

since the modifications are not very great in the case of a stable shell, the approximate theory may be considered sufficient in the present discussion.

(b) Effect of a Displacement of the Principal Axis of Inertia.

If the principal axis of inertia C makes a small angle with the axis of figure, the effect is similar to that of an impressed impulsive moment. Let δ be the angle between the axes. Up to the instant of ejection the projectile is forced to rotate about its axis of figure, its angular velocity at ejection being ω_o. The component angular velocities along the principal axes are $\omega_o \cos \delta$ and $\omega_o \sin \delta$, and the angular momenta along these axes are $C\omega_o \cos \delta$ and $A\omega_o \sin \delta$ respectively. The total angular momentum is found by compounding these as in the diagram (Fig. 13).

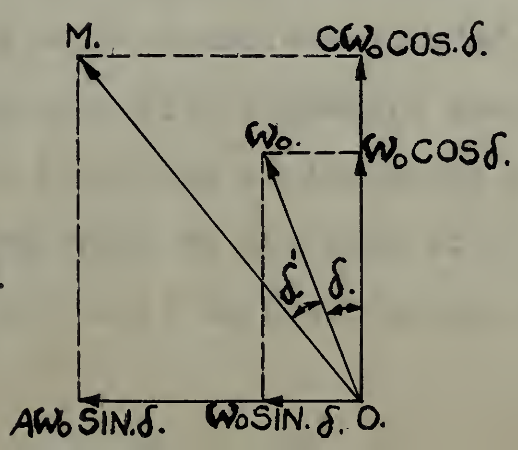

FIG. 13.

The momentum axis therefore makes with the instantaneous axis ω_0 an angle (δ') which will be opposite to δ when A $>$ C. Since after ejection the angular momentum remains constant, the figure may as before be considered to rotate about the axis O M and the motion of the projectile represented by that of a cone of amplitude δ rolling on a fixed cone of amplitude δ'. The vector diagram is precisely like that of the previous case except that it is turned ~~rotated~~ through the angle δ, and ω_0 takes the place of ω. (Compare figs. 12 and 13.) The principal axis of inertia C will rotate about the vector O M with the period

$$T = \frac{2\pi A \cos(\delta + \delta')}{C\omega_0 \cos\delta}$$

which is for small angles practically equal to the result in the previous case.

There are two points of difference from the former case. The first is that the deviation of the axis of momentum, and hence the average displacement of the principal axis from the line of fire is now not $\delta + \delta'$ but δ', where

$$\delta' \doteq \frac{A - C}{C}\delta$$

The other is that, while as before the principal axis of inertia C describes a cone around OM, the axis of figure of the projectile now coincides not with the axis of the rolling cone but with the element of the cone which coincided with ω_0 at the instant of ejection. This axis will therefore describe an epicycloidal cone, the amplitude of the loops being δ on either side of the cone described by the axis of C.

In the case of the 14 inch projectile where

$$A/C = 6$$

$$\delta' = 5\delta$$

Therefore to produce a deviation averaging 1 degree on one side of the line of fire, the principal axis of inertia must be tilted 0.2 degree from the axis of figure.

The effects of the various causes enumerated above will now be considered in turn.

(1) Vibrations of the Gun

From records taken of the motion of the muzzle of the gun in recoil, the amplitude of the muzzle vibration is apparently less than one millimeter. Assume,

Amplitude = 1/25 inch

Frequency = 100

Muzzle velocity = 2800 ft./sec.

Then the maximum velocity would be

$$\frac{2\pi \times 100}{25 \times 12} = 2 \text{ ft./sec.}$$

This is the lateral velocity which would be given to the shell if it emerged when the muzzle was passing the equilibrium position.

The direction of flight might then be shifted by about $\frac{2}{2800}$ or about 2.5'. If the shift was in the vertical plane, it would produce a maximum change of range of about $\pm$ 43 yards in a range of 25,000 yards.

In addition to this lateral velocity there would be impressed upon the shell on ejection the angular velocity of the gun at that instant. This would produce the effect dis-

cussed under (a) above. If the ejection occurred at the time of maximum velocity, it might be estimated as $\omega' = \frac{2}{50}$ or $\frac{1}{25}$ radians per sec. (taking 50 ft. for the effective length of the gun). This rotation about a transverse axis would tilt the axis of momentum by an angle

$$\delta + \delta' = \frac{A}{C}\,\frac{\omega'}{\omega_0}$$

where A and C are the principal moments of inertia of the shell and ω_0 its angular velocity of spin. Since

$$\frac{A}{C} = 6 \qquad \omega_0 = 603$$

we have

$$\delta + \delta' = \frac{6}{25 \times 603} = .0004 = 1.4'$$

The oscillations would therefore be of very small amplitude.

It may therefore be concluded that unless the vibrations are considerably greater than estimated, their disturbing effect may be neglected in the case of the 14 in. shell. However, it is very desirable to obtain more reliable data on this point. But little work has been done, and that on rifle barrels. See Cranz Vol.I, p. 328; also C. Cranz & K.R.Koch, Munch.Akad.Ber.21 (1901) p. 572; C. Cranz, Sitzber. Berlin. Math. Ges. 3 (1904) p 11.

(2) Jump and Whip of the Gun

In the Bureau of Standards report on the tests of the firing of the U.S.S. Mississippi in 1918, the jump and whip were investigated. The maximum effect observed at the instant of ejection of the shell was a jump of 1.2 ft. per sec. with an angular displacement of 5.5 minutes. These gave a

variation in range of 98 yards. (See Table IV of the report). The average deflection was much less than this.

In addition to the effects considered in the report, the angular velocity of the muzzle would be communicated to the shell as in the case of the vibrations, producing a similar effect. As the maximum observed velocity is about half that estimated as due to vibrations, it is evident that in this case also the disturbing effect is very small.

(3) Blast from a Neighboring Gun

There is available as yet little information on the magnitude of the forces in the blast.* An estimate of the

* The blast from a rifle has been studied by M. Okochi, Pro. Toky. Math. Phys. Soc. (2) 7 (1913) 104-114 and by C. Cranz & B. Glatzel, Ann. Phys. 43 (1914) 1186-1204.

possible effect on a shell of the blast from a neighboring gun has been made from a study of a series of photographs taken of the discharge of a 12 inch gun fired with a muzzle velocity of 2200 feet per second.

This study made by Dr. F. E. Kester gave the following data which will serve at least to give approximate values.

Longitudinal diameter of blast	-	15 ft.
Transverse diameter of blast	-	22 ft.
Longitudinal velocity of blast	-	630 ft./sec.
Transverse velocity of blast	-	500 ft./sec.

Using these data, it is possible to calculate the effect of a projectile traveling through the blast assuming the whole effect to be lateral. The force may be estimated by the formula for air resistance (See p. 4)

$$R = S \cdot \rho \cdot i \, f(v)$$

ρ being the relative density of the gas, i the form factor, S the area, and f(v) the tabulated force per unit area on standard shell at standard density.

The density of the gas is estimated by dividing the charge by the volume of the cloud which may be assumed an oblate spheroid of diameters 15 and 22 ft. The volume is therefore

$$V = \frac{4}{3}\pi . 11 . 11 . \frac{15}{2} = 3800 \text{ cu.ft.}$$

The normal charge is 270 lbs. Assuming that 2/3 of the charge was in the blast, the density of the gas would be

$$\frac{180}{3800} = .047 \text{ lbs. per cu. ft.}$$

or nearly 2/3 the density of air. This seems reasonable, for M. Okochi, working with a rifle blast closer to the muzzle obtained values two or three times the density of air.

The form factor i of the projectile viewed laterally may be estimated at 3/2. That of a hemisphere is 1.35, while the form factor of the projectile end on is 0.70.

The cross-section of a 14 inch projectile is about 580 sq.in.

The value of f(v) ~~taken~~ calculated from Krupp's tables (Cranz Vol. ~~II~~ I p 58) for 152 m/sec. = 500 ft./sec. is

$$.027 \text{ kg/sq.cm.} = 0.385 \text{ lbs./sq.in.}$$

We may then calculate the transverse force on a 14 inch shell passing through the blast as

$$R = \frac{2}{3} \times \frac{3}{2} \times 580 \times 0.385 = 223 \text{ lbs.}$$

The time during which the force acts is that during which the shell travels through the blast or $\frac{15}{2800} = .005$ sec. The transverse velocity produced would therefore be

$$v = \frac{R\ t}{m} = \frac{223 \times .005 \times 32}{1400} = .025 \text{ ft./sec.}$$

The effect of a blast of a 14 in. gun may be estimated as about twice this.

To estimate the effect upon the axis of the shell, we may assume the center of pressure to be 3 inches from the center of gravity.

The moment of the impulse will then be

$$M' = 223 \times \frac{3}{12} \times .005 = 0.29 \text{ lb.ft.sec.}$$

This moment would tilt the axis less than $1/4'$. While there is much uncertainty as to the data assumed, there seems no reason to doubt that the results give the order of magnitude of the effect. It may therefore be inferred that the blast of a neighboring gun has a very small effect upon a shell.

(4) Irregular Action of Powder Gases and Direct Action of Blast

On this point there is no information except that it is shown by photographs that the gases sometimes escape first from one side and the blast is consequently unsymmetrical. The effect may be estimated by assuming that the center of pressure is displaced one inch during the time in which the shell travels the last two feet of its path in the gun. The moment of the impulse will be

$$M' = \int L\,dt = F\,r\,t = P\,S\,r\,t$$

where P = the pressure of the powder gases = 10000 lbs. per square inch

$S = \pi d^2/4 = 154$ sq.in.

$r = 1/12$ ft.

$t = \dfrac{2}{2800}$ sec.

or

$$M' = \frac{154 \times 10^4 \times 2}{12 \times 2800} = 92 \text{ ft.lb.sec.}$$

This would produce an oscillation of a little over 1 degree amplitude.

After the shell emerges, the blast would continue to act unsymmetrically, but its effect would rapidly diminish. It seems therefore that a considerable want of symmetry would be necessary to produce appreciable oscillations. Further evidence of the magnitudes of the quantities here involved seems very desirable.

Even if the blast were symmetrical, it would produce an acceleration of the shell, and also, if the shell was tilted from some other cause, a rotating couple. There is evidence, somewhat conflicting, of the acceleration. The magnitude of this effect cannot be estimated until more data are available.

(5) <u>Lateral Displacement of the Center of Gravity</u>

If the center of gravity of a shell is displaced laterally a distance _r_ but the shell is otherwise balanced, the center of gravity while the projectile is in the gun will move in a helix of radius _r_; when it leaves the gun it continues to

move along the tangent to the helix, and therefore in a direction which may be found by compounding the muzzle velocity v with a transverse velocity $\omega_0 r$.

There may also be a tilting moment at the time of ejection, beginning to act when the bourrelet is free. Thus if $\bar{X}\ \bar{Z}$ are the coordinates of the center of gravity measured

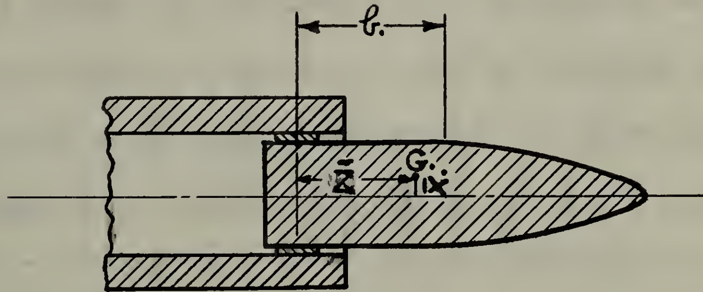

FIG. 14.

from the center of the rotating band (Fig. 14), there will be, due to the rotation, a couple $\frac{W}{g}\omega_0^2\ \bar{X}\ \bar{Z}$ acting in nearly a constant direction during the time between the emergence of the bourrelet and that of the band. In addition, if the center of pressure of the powder gases is at the center of the base, this will produce a couple of magnitude $P\ S\ \bar{X}$ in the same direction, P being the pressure of the powder gases, S the area of the base of the shell. These couples are more or less neutralized by one due to the rigidity of the rotating bands L_r. The resultant couple will then be

$$L = \frac{W}{g}\omega_0^2 \left(\bar{X}\ \bar{Z} + P\ S\ \bar{X} - L_r \right.$$

and the impulse moment

$$M' = \int L\,dt = \left(\frac{W}{g}\omega_o^2 \bar{X}\bar{Z} + P S \bar{X} - L_r\right) t$$

where t, the time during which the couple acts, is given by

$$t = \frac{b}{V_o}$$

b being the distance between bourrelet and band.

These expressions may now be applied to the 14 inch shell, in order to estimate the probable effect of the cavity being eccentric. The net loss of weight due to the cavity being filled with charge instead of iron has been calculated to be about 140 lbs.* or 1/10 the total weight of the shell.

---

* See appendix.

Therefore a shift of the cavity of .05 inch would shift the c.g. by only .005 inch. This estimate seems excessive, and it is probable that a blow hole in the metal of sufficient size to produce an equal shift would be detected by the shell being below normal weight.

Assuming $\bar{X}$ = .005 in., the transverse velocity imparted will be given by

$$V = \omega_o \bar{X} = 603 \times .005 \times 1/12 = 0.25 \text{ ft./sec.}$$

which would change the direction of flight by only 0.3'.

To calculate the tilting effect, we may assume

$\bar{X}$ = .005 in.	P = 10000 lb./sq.in.
$\bar{Z}$ = 16 in.	S = 154 sq. in.
b = 22 in.	W = 1400 lbs.
V_o = 2800 ft./sec.	L_r = 0
ω_o = 603	

from which

$$t = \frac{22}{12 \times 2800} = .00065 \text{ secs.}$$

$$M' \int L\, dt = \left(\frac{1400}{32} 603^2 \times \frac{.005}{12} \times \frac{16}{12} + 10000 \times 154 \times \frac{.005}{12}\right) .00065$$

$$= (8840 + 642)\ .00065$$

$$= 6.2 \text{ lb.ft.sec.}$$

Since it would require 79 lb.ft.sec. to produce an oscillation of the axis of 1 degree amplitude, the effects here considered would produce an amplitude of less than 5'.

It is therefore not to be expected that the displacement of the center of gravity will have great effect on the motion of the shell in normal cases.

(6) <u>Angular Displacement of the Principal Axis</u>

The effect of an angular shift of the principal axis of inertia has been discussed under (b) above (p.32). To apply these results to the present problem the amount of the shift will be calculated on the assumption that a given mass is added to (or subtracted from) a symmetrical projectile.

Let M', A_0, B_0, C_0 be the mass and principal moments of inertia of the shell ($A_0 = B_0$); m the added mass.

The axis of the shell is taken as the axis of Z, the plane through the added mass as that of X Z (the origin being at the original center of gravity of the shell), and the coordinates of the added mass as $X_1\ 0\ Z_1$. (Fig. 15). The center of gravity is shifted to the point G' whose coordinates

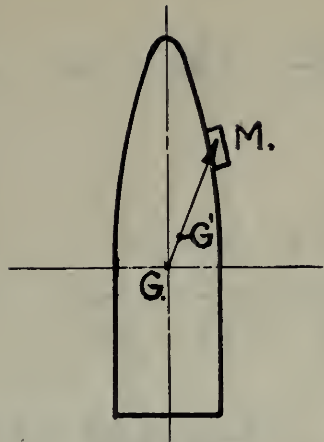

FIG. 15.

are $\quad \bar{X} = \frac{m}{M + m} X_1 \; ; \quad 0; \quad \bar{Z} = \frac{m}{M + m} Z_1$

The moments and products of inertia about parallel axes through the new center of gravity must now be determined. Let them be denoted by A, B, C; D, E, F respectively. Then

$$A = A_o + m Z_1^2 - (M + m) \bar{Z}^2 = A_o + \frac{M m Z_1^2}{M + m}$$

$$C = C_o + m X_1^2 - (M + m) \bar{X}^2 = C_o + \frac{M m X_1^2}{M + m}$$

$$D = F = 0 \text{ by symmetry}$$

$$E = 0 + m X_1 Z_1 - (M + m) \bar{X} \bar{Z} = \frac{M m X_1 Z_1}{M + m}$$

Now the moment of inertia about an axis in the plane of Z X making an angle δ with the axis of Z is from the property of the ellipsoid of inertia.

$$C' = A \sin^2\delta + C \cos^2\delta - 2 E \sin\delta \cos\delta$$

If this is the principal axis, it is a minimum (or maximum) so

$$\frac{\delta C'}{\delta \delta} = 0 = 2 A \sin\delta \cos\delta - 2 C \sin\delta \cos\delta - 2 E (\cos^2\delta - \sin^2\delta),$$

i.e. $\tan 2\delta = \dfrac{2E}{A-C}$

Substituting

$$\tan 2\delta = \frac{2\,\dfrac{Mm\,X_1Z_1}{M+m}}{A_o - C_o + \dfrac{Mm}{M+m}\left(Z_1^2 - X_1^2\right)}$$

If the ratio m/M is small $\dfrac{M}{M+m} \doteq 1$

So that approximately

$$\delta \doteq \tan\delta \doteq \frac{m\,X_1\,Z_1}{A_o - C_o}$$

If for instance the asymmetry is produced in a 14 inch shell by a 5 pound piece of metal (perhaps a piece of the rotating band) being removed from a point on the edge of the base of the projectile ($X_1 = -\dfrac{7}{12}$ $Z_1 = -\dfrac{19}{12}$) the tilt of the axis ~~is found to~~ will be

$$\delta = \frac{\frac{5}{32}\cdot\frac{7}{12}\cdot\frac{19}{12}}{45 - 7.5} = .000385 = 13'$$

and a tilt of this amount it has been shown above would produce an oscillation of amplitude less than 1 degree. As it is unlikely that an accidental want of symmetry of the projectile would be at all comparable with that assumed, it is probable that oscillations due to this cause would be of very small magnitude.

PART III. THE DECREASE OF SPIN OF A RIFLED PROJECTILE IN FLIGHT

As has been stated above (p.18), but little information is available concerning the decrease in spin of a projectile during its flight. The theory and the experimental data are summarized in the following paragraphs.

Theory

Consider a projectile of diameter d, length ℓ, moment of inertia C, rotating with angular velocity ω. The frictional moment retarding the spin may be assumed proportional to

(1) the relative velocity of the surface, $\propto \omega d$

(2) the area " " " , $\propto \ell d$

(3) the radius " " " , $\propto d$

Therefore

$$L_F \propto \ell d^3 \omega$$

But for similar projectiles $C \propto \ell d^4$, so that

$$L_F = -k \frac{C}{d} \omega \; ;$$

and as

$$L_F = C \frac{d\omega}{dt}$$

it follows that

$$\frac{d\omega}{dt} = -\frac{k\omega}{d}$$

The coefficient k will vary with the form of the shell and probably also with its velocity and its tilt. As a first approximation, k may be assumed constant. For a value which

would be probably more accurate it might be taken proportional to the air resistance per unit area.

If k is constant, the spin at any time will be

$$\omega = \omega_0 e^{-kt/d} *$$

---

* This result is equivalent to that obtained by Röggla, who gives the formula

$$\omega = \omega_o e^{-c \nu d^4 t/C}$$

the letters having the same meaning and ν standing for the ratio ℓ/d.

Greek nu

If k decreases with the velocity, the spin falls off more slowly than the formula indicates.

The total number of revolutions in a given time t is obtained by integrating the expression for ω and is found (when k is constant) to be

$$n = \frac{\omega_o d}{2\pi k} (1 - e^{-\frac{k}{d}t})$$

or if $\frac{\omega_o}{2\pi} = A$, the initial rate of spin in revolutions per second,

$$n = A \frac{d}{k} (1 - e^{-\frac{k}{d}t})$$

Experimental Data

The general view has been that the change of spin is very small (see Rohne 1911, quoted by Cranz III,285). There have been few published results. These may be summarized as follows:

(1) F. Neesen has been experimenting since 1902 on the ballistic data of projectiles, using a projectile with a

magnesium flame on one side and photographing the traces which appear as dotted lines because of the spin.* He

---

* For description of shell see Kriegst. ZS. 1903 p. 112

---

gives few results on the rotation and these are somewhat indefinite. As far as they can be used to determine k, they are as follows:

(a) In 1902 Neesen gives data for two shells of 8.8 cm caliber. He finds that after traveling 1340 meters they move2.935 and 2.905 meters respectively during one rotation. Using ballistic tables which give a final velocity of 300 meters per second, he calculated a spin of 102.7 revolutions per second. From the initial velocity assumed, the initial spin was 110.

The time of flight is not given but using ballistic tables and probable values for the constants it may be estimated as 3.67 seconds.** Using these values, we have

---

** Neeson states that the shell was "a heavy field shell 88" (Feldgranate). The data for this are given by Heydenreich (II pp 93 & 157).

d = .088 m		Initial vel.	= 442
ℓ = 2.6 cal		Rot.per sec.	= 100
W = 7.45 kg		Rifling	= 1 to 50 cal
	Form factor about 1.155		

The shell was therefore probably fired with a muzzle velocity 442 x 110/100 = 486 meters per sec. From these data and the given final velocity of 300 meters per sec. the time of flight is estimated as 3.67 secs.

$$102.7 = 110\, e^{-\frac{k}{d} 3.67}$$

whence $\frac{k}{d} = .0187 \left(\begin{array}{l} k = 0.165 \text{ cm/sec} \\ = 0.065 \text{ in/sec} \end{array} \right.$

This result is very uncertain because of the various assumptions necessary to obtain it.

(b) In 1906, he gives results obtained from a 16 kg shell of 11 cm caliber fired from a Krupp howitzer with 25 calibers rifling. The data given* are as follows:

---

* Some of these data are supplied by Heydenreich Vol. II, p. 146.

Range	Initial Velocity	Final Velocity Ob-served	Final Velocity From Tables	Angle of Fall Ob-served	Angle of Fall From Tables	Initial Rota-tion	Final Rota-tion
1550 m	161 m/sec	144	(146)	20°	20°17'	59.1	55.6
1550	304.1 "	270	(262)	5°10'	5°44'	110.6	107.1
3000	304.1 "	230	(232)	12°40'	13°44'	110.6	99.5

From these we may by ballistic tables estimate

Angle Departure	Time Flight	Coefficient Form
19° 36'	10.8 sec.	1.9
5° 15'	5.5 "	1.7
11° 34'	11.7 "	1.7

(The large value of the coefficient of form indicates a blunt or rather unsteadyprojectile).

Using these values of time, we obtain

	k/d	k(cm/sec)	k(in/sec)
1.	.00565	.0622	.0245
2.	.00588	.0647	.0254
3.	.00950	.1045	.0412

NOTE: Heydenreich gives somewhat different data. In particular 100.3 for the final velocity in case 3. This would give for the last line

3' .00837 .0921 .0363

Neesen has continued his work but no more data are at present available.

(2) Bethell in " Modern Guns and Gunnery" (1910) states (giving no details) that by "recent experiments with mechanical fuses depending for their action on the spin of the shell.....it is found that a Q.F. field shell loses about 10% of its spin at 3000 yards and 20% at 5000 yards." The shell was probably an 18 pounder (3.3 in. caliber). Again he states: "It is now considered that the 15 pounder loses 20% of its velocity of rotation at 3000 yards and 30% at 5000 yards. Shells which are steadier in flight and of larger caliber, such as the 18 pounder, lose their velocity of rotation at about half this rate."

In the absence of further data, it is not possible to determine k from this with any certainty. It is probable that these results are from a preliminary report of the experiments of Hill (discussed below) since in a revision of 1913 Bethell substitutes Hill's results. It is therefore unnecessary to attempt to estimate k from these data.

(3) In 1911 Captain H. W. Hill published results on the loss of spin of shrapnel. The method was to use a mechanical fuse which exploded the shell after a definite number of revolutions, and to observe the time of burst and range.

His principal series of experiments was made with an 18 pounder gun (3.3 in caliber) and the results (given in the table) are the means of 18 rounds at each range.

Number Revolutions	Range (Yards)	Time to Burst (Seconds)
1200	2631	6.70
2400	4703	14.04
3700	6675	22.48

The initial spin was not measured but calculated from the assumed initial velocity of 1590 feet per second and rifling pitch of 30 calibers to be 192.5 revolutions per second. This is doubtful, as the gun was stated to be an old one. With a 15-pounder (caliber 3 in.) he made a single series as follows:

Number Revolutions	Range (Yards)	Time to Burst (Seconds)
3200	4963	16.98

Initial spin was calculated from initial velocity of 1581 and rifling pitch of 28 calibers to be 226 but was assumed to be 223 to allow for " over-ride" .

In reducing his results Hill uses ranges instead of times and deduces the percentage loss of spin as function of the range. It seems preferable to discuss them using time as the variable.

(a) The 18 pounder experiments.

As the data giving the initial spin are doubtful it may be considered unknown. Using the equation obtained

above (p.46) $$n = A \frac{d}{k} (1 - e^{-\frac{k}{d} t})$$

A and k/d are determined from each pair of observations.

The first and second lines of the table give the result

$$A = 187, \quad \frac{k}{d} = .013$$

So that

$$\omega = 2 \pi \, 187 \; e^{-.013 t}$$

The second and third give

$$A = 182, \quad \frac{k}{d} = .009$$

So that

$$\omega = 2 \pi \, 182 \; e^{-.009 t}$$

The values for the initial spin agree within 3% but are considerably different from that assumed by Hill. The results indicate that value of k, and hence the rate of loss of spin diminished with time, as is suggested by theory. The experiments, however, are hardly sufficient to show this. Thus if it is assumed that

$$A = 183.8; \quad \frac{k}{d} = .010$$

So that

$$N = 18380 (1 - e^{-.01 t})$$

We obtain

t	N
6.70	1191.1
14.04	2407.6
22.48	3700.3

HILL'S EXPERIMENTS ON LOSS OF SPIN.

+ OBSERVATIONS.

—O— $N=18350(1-e^{-.01t})$.

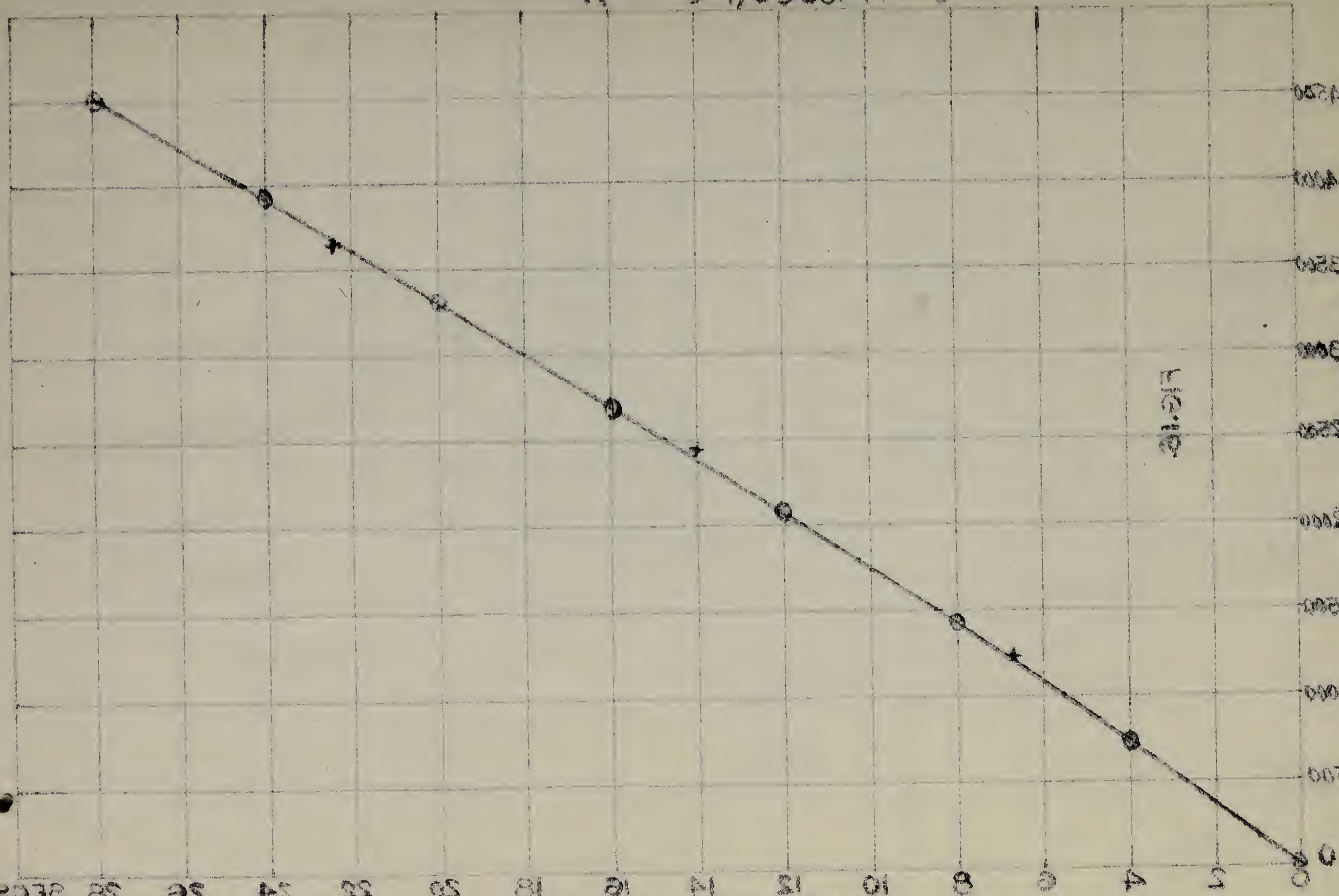

FIG.16.

NO. REVOLUTIONS.

TIME OF FLIGHT.

Which is probably within the limits of experimental error. (See curve Fig. 16)

The conclusion from these experiments is that $\frac{k}{d}$ is about .01, which would give

$$\begin{cases} k = .033 \text{ inches / second} \\ k = .084 \quad \text{cm / second} \end{cases}$$

It should be added that Hill's results were criticized by H. J. Jones, who pointed out the inaccuracy of the data. Jones gives an empirical formula to represent the data, from which he infers an initial spin of 178.4 revolutions per second. This is probably too low.

(b) <u>The 15 pounder experiments</u>

Here data are given for but one range. It is necessary to know the initial spin. If as Hill assumes this was 223 revolutions per second, the value of k/d is given by

$$3200 = 223 \frac{d}{k} \left(1 - e^{-\frac{k}{d} 16.98}\right)$$

the solution of which is

$$\frac{k}{d} = .0204; \quad k = .0612 \text{ in/sec} = .156 \text{ cm/sec}$$

If on the other hand, the initial spin is taken as much below its theoretical value as with the 18 pounder, its value would be

$$226 \times \frac{184}{192.5} = 216$$

and we should obtain

$$\frac{k}{d} = .0165; \quad k = .0495 \text{ in/sec} = .126 \text{ cm/sec}$$

The results indicate the order of magnitude to be the same as for the 18 pounder.

Summary of Results

	d(cm)	V_0(m/sec)	t	k/d	k(cm/sec)	k(in/sec)
Neesen 1902	8.8	484	3.67	.019	.165	.065
Neesen 1906	11.0	161	10.8	.0057	.062	.025
" "	11.0	304	5.5	.0059	.065	.025
" "	11.0	304	11.7	.0095	.105	.041
Hill 1911	8.4	485	22.5	.010	.084	.033
" "	7.6	482	17.0	.020	.156	.061

Of these results the first and last are least accurate and the best are probably those of Hill with the 8.4 cm shell. We may estimate k as therefore about .08 or .09 cm/sec for average shells of from 7.5 to 11 cm caliber. We have no information as to how far we may extrapolate these values for larger or smaller projectiles or for different velocities.

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APPENDIX

Calculation of Moments of Inertia of a 14-inch Shell

In order to obtain an estimate of the moments of inertia of the shell a simplified body was assumed consisting of a semi-ellipsoid mounted on a cylinder with a cavity of the same description uniformly filled with the charge, and an added copper ring and pointed tip. The moments of inertia of this body were calculated about a point on the axis 19.3 inches from the base, as this corresponds to the center of gravity of the actual shell. As a check a calculation was made of the position of the center of gravity of the assumed body.

Figure 17 gives the essential dimensions of the actual shell as given by the specifications, and Figure 18 those assumed for the simplified body. (All distances are given in inches.) In Figure 18

E_1 indicates the center of gravity of the outer semi-ellipsoid.

E_1 indicates the center of gravity of the outer cylinder.

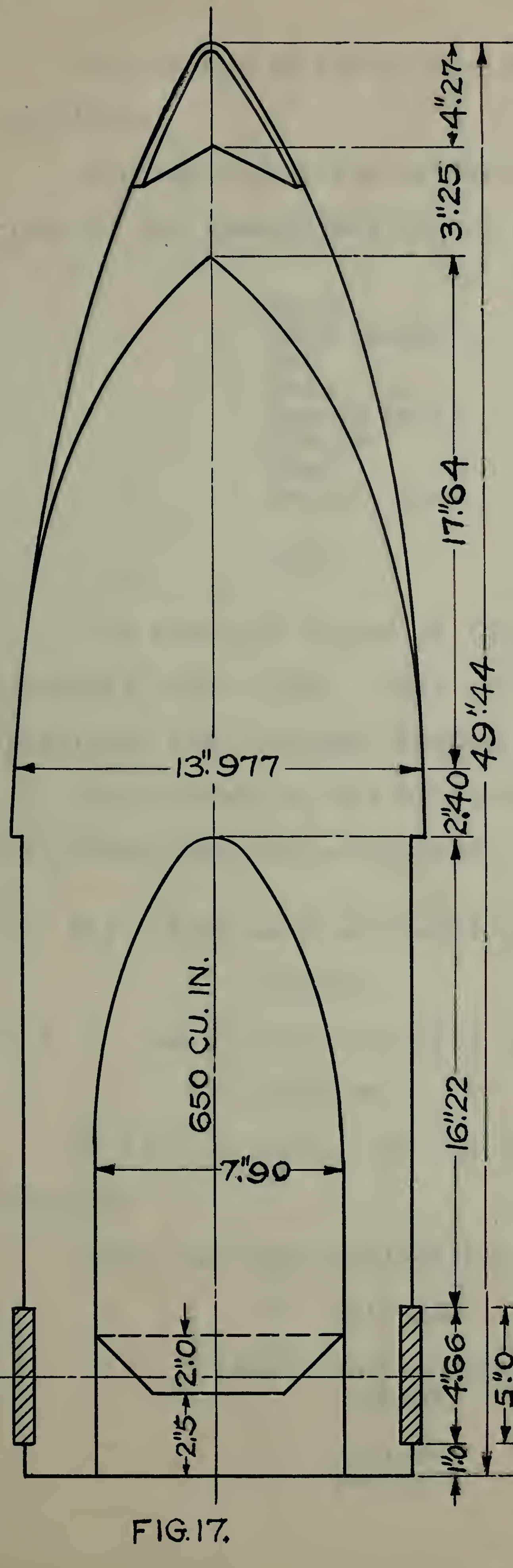
E_2 indicates the center of gravity of the inner semi-ellipsoid.

C_2 indicates the center of gravity of the inner cylinder.

R indicates the center of gravity of the ring.

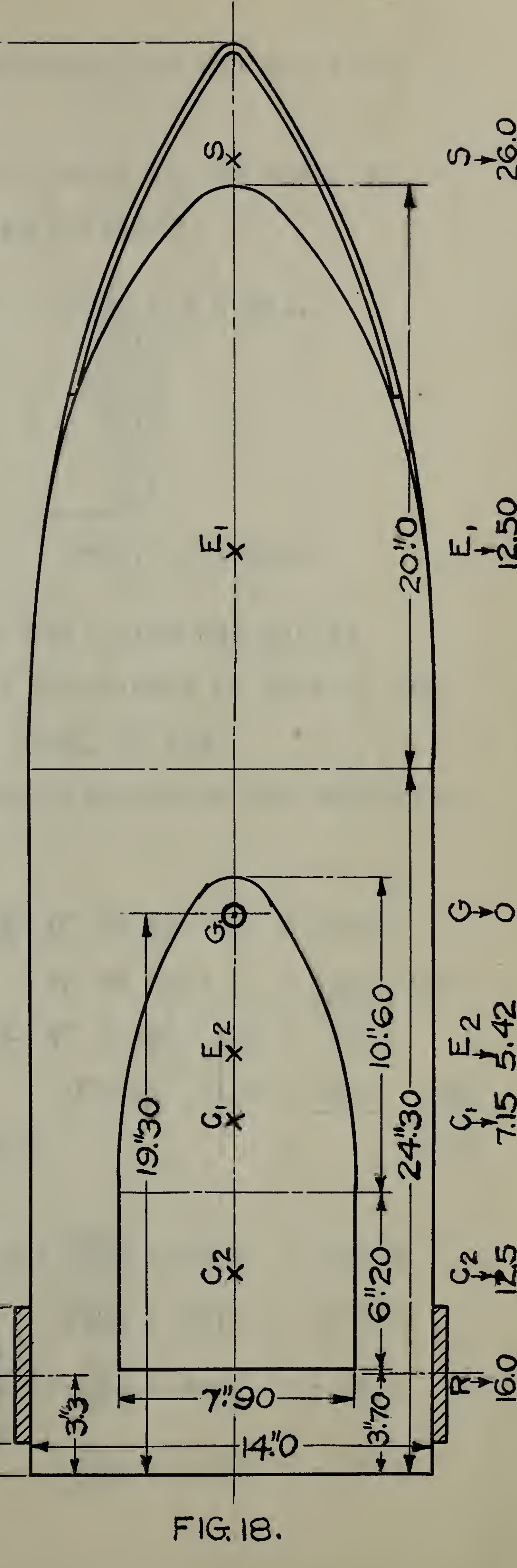
S indicates the center of gravity of the tip.

G indicates the assumed center of gravity.



4".27
3".25
17".64
49".44
13".977
2".40
16".22
650 CU. IN.
7".90
2".0
4".66
5".0
2".5
1".0

FIG.17.



S
S 26.0
20".0
E1
E1 12.50
G
G 0
10".60
19".30
E2
E2 5.42
G1
G1 7.15
24".30
G2
G2 12.5
6".20
7".90
R 16.0
3".3
14".0
3".70

FIG.18.

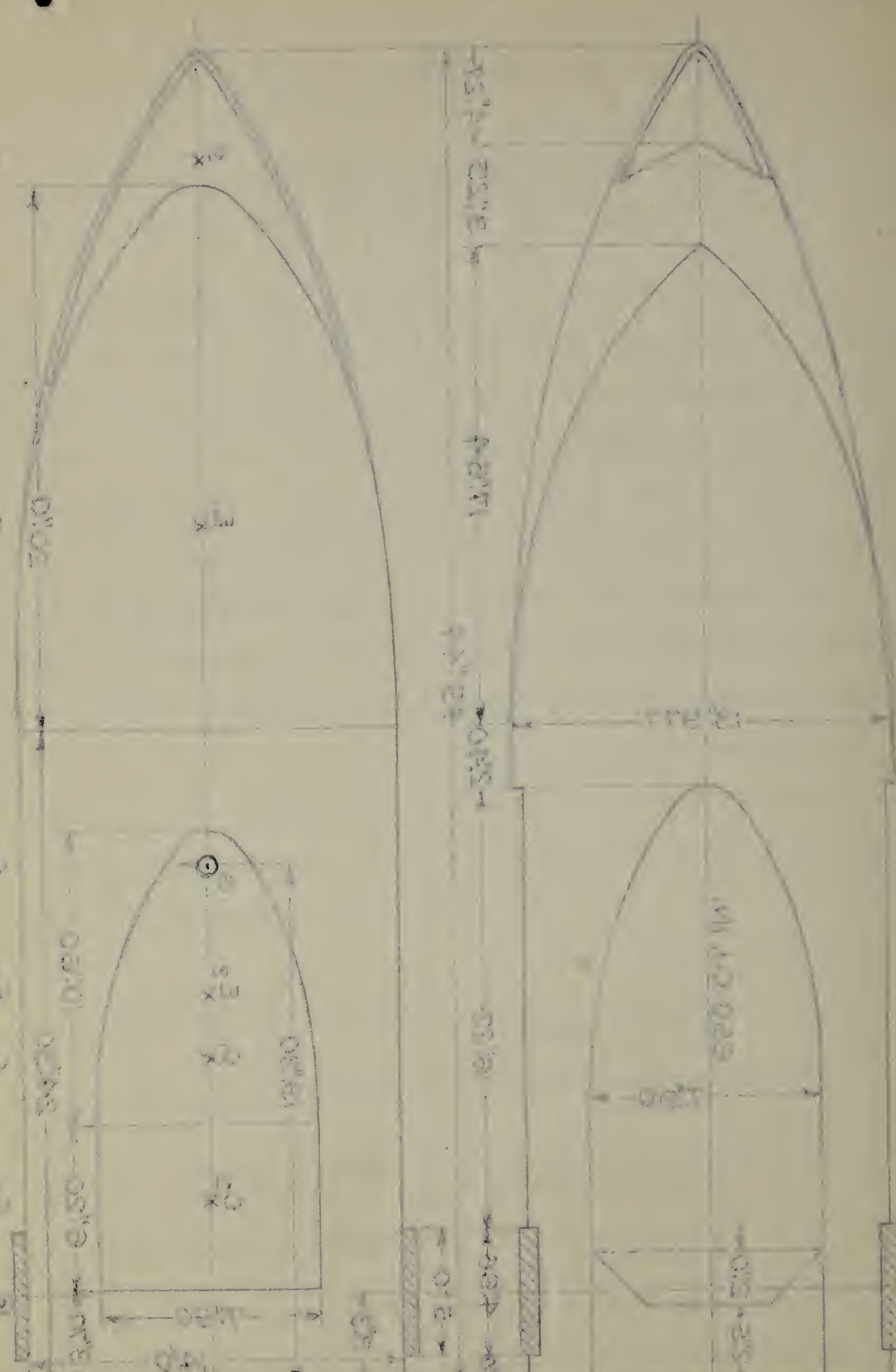


FIG. 16

FIG. 17

The distances from G are indicated on the margin of the figure.

The weights of the different parts of the shell as given in the specifications are as follows:

Shell	1196.5 ± 4 lbs.
Wind Shield	5.8
Cap	84.1
Base Plug	51.2
Copper Band	29.4
Charge	29.5
Fuse	3.0
Gasket, etc.	0.5
Total	1400.0 ± 4 lbs.

The combined weight of the shell plug and cap is therefore 1335.3 lbs. This will correspond to that of the simplified shell without charge, ring, or tip.

The weights of the different elements of the simplified shell were obtained as follows:

Vol. large semi-ellipsoid	$\frac{2}{3}\pi\ 7^2\ 20$	=	2055	
" " cylinder	$\pi\ 7^2\ 24.3$	=	3745	5800
Less " small semi-ellipsoid	$\frac{2}{3}\pi\ 3.95^2\ 10.6$	=	346	
" " cylinder	$\pi 3.95^2\ 6.2$	=	304	650
So. Vol. of shell, cap and plug		=		5150

Therefore

Weight of large semi-ellipsoid	$\frac{2055}{5150} \times 1335$	=	532.8
" " " cylinder	$\frac{3745}{5150} \times 1335$	=	971.0
" " small semi-ellipsoid cavity	$-\frac{346}{5150} \times 1335$	=	- 89.7
" " " cylinder cavity	$-\frac{304}{5150} \times 1335$	=	- 78.8

Weight of charge in ellipsoid $\frac{346}{650} \times 29.5 = 15.7$

" " " " cylinder $\frac{304}{650} \times 29.5 = 13.8$

So net weight of small ellipsoid $= -74.0$

" " " " " cylinder $= -65.0$

The C. G. of a semi-ellipsoid is $\frac{3}{8}\,a$ from center.

The longitudinal (rad. gyr.)^2 is $\frac{2}{5}\,b^2$

The transverse (rad. gyr.)^2 through center is $\frac{1}{5}\,(a^2 + b^2)$

$\therefore$ transverse (rad. gyr.)^2 through C. G. is $\frac{1}{5}\,b^2 + \frac{19}{320}\,a^2$

Here a and b are the longitudinal and transverse semi-axes.

For each figure was calculated

$\bar{x}$ the distance of C. G. from base (~~or~~ also the distance from G.)

k_1^2 the (rad. gyr.)^2 about axis of figure.

k_2^2 the (rad. gyr.)^2 about perpendicular axis 19.3 in. from base (G).

(1) Ell.$_1$ $\bar{x} = \frac{3}{8}\,20 + 24.3 = 31.8$ from base (or 12.5 from G.)

$$k_1^2 = \frac{2}{5}\,7^2 = 19.6 \text{ sq.in.}$$

$$k_2^2 = \frac{7^2}{5} + \frac{19}{320}\,(20^2) + (12.5)^2$$

$$= 9.80 + 23.75 + 156.25 = 189.8 \text{ sq.in.}$$

(2) Cyl.$_1$ $\bar{x}$ = 12.15 from base (or 7.15 from G.)

$$k_1^2 = \frac{1}{2} 7^2 = 24.5 \text{ sq.in.}$$

$$k_2^2 = \frac{1}{4} 7^2 + \frac{(24.3)^2}{12} + (7.15)^2$$

12.25 + 49.20 + 51.12 = 112.6 sq.in.

(3) Ell.$_2$ $\bar{x} = \frac{3}{8} 10.6 + 9.9 = 13.88$ from base (or 5.42 from G.)

$$k_1^2 = \frac{2}{5} \frac{(7.9)^2}{4} = 6.24 \text{ sq.in.}$$

$$k_2^2 = \frac{1}{5} \frac{(7.9)^2}{4} + \frac{19}{320} (10.6)^2 + (5.42)^2 \text{ from } 19.3$$

3.12 + 6.67 + 29.38 = 39.2 sq.in.

(4) Cyl.$_2$ $\bar{x}$ = 3.1 + 3.7 = 6.8 from base (or 12.5 from G.)

$$k_1^2 = \frac{1}{2} \frac{(7.9)^2}{4} = 7.80 \text{ sq.in.}$$

$$k_2^2 = \frac{1}{4} \frac{(7.9)^2}{4} + \frac{(6.2)^2}{12} + (12.5)^2$$

3.90 + 3.20 + 156.25 = 163.4 sq.in.

(5) Ring $\bar{x}$ = 3.3 from base (or 16.0 from G)

$$k_1^2 = 7^2 = 49 \text{ sq.in.}$$

$$k_2^2 = \frac{7^2}{2} + \frac{5^2}{12} + 16^2$$

24.50 + 2.08 + 256. = 282.6 sq.in.

(6) Tip. This may be considered with sufficient accuracy as a mass concentrated at 45.3 from base (or 26.0 from G.)

$$k_1^2 = 0$$

$$k_2^2 = 26^2 = 676.\ \text{sq.in.}$$

	W	x	k_1^2	k_2^2	Wx	Wk_1^2	Wk_2^2
Large Ell.	532.8	31.80	19.6	189.8	16940	10440	101130
Large Cyl.	971.0	12.15	24.5	112.6	11800	23790	109340
Small Ell. Net	-74.0	13.88	6.24	39.2	-1030	-450	-2900
Small Cyl. Net	-65.0	6.80	7.80	163.4	-440	-510	-10620
Ring	29.4	3.30	49.0	282.6	100	1440	8320
Tip	5.8	45.30	--	676.0	260	--	3920
Total	1400.0				27630	34710	209190

Whence C. G.

$$\bar{x} = \frac{27630}{1400} = 19.7 \text{ in. which checks fairly.}$$

Longitudinal Mom. Inertia

$$k_1^2 = \frac{34710}{1400} = 24.8 \text{ sq.in.}$$

$$C = \frac{34710}{144 \times 32.17} = 7.50 \text{ slug - sq.ft.}$$

Transverse Mom. Inertia

$$k_2^2 = \frac{209190}{1400} = 149.4 \text{ sq.in.}$$

$$A = \frac{209190}{144 \times 32.17} = 45.2 \text{ slug - sq.ft.}$$

$$\text{Ratio } \frac{A}{C} = \frac{45.2}{7.50} = 6.03$$

REPORT OF OBSERVATIONS MADE BY H. L. CURTIS ON A TRIP TO FRANCE AND ENGLAND DURING APRIL, MAY, AND JUNE 1919.

This trip was undertaken primarily to obtain information concerning the developments in ballistic instruments which have been made in England and France during the war. However, observations were not confined to this subject alone but a number of other subjects of interest to this Section and other Sections of the Bureau were investigated.

Reports were forwarded to the Bureau of Standards from day to day concerning all points of interest visited and all information of value was transmitted. These fragmentary reports serve as the basis for this report which will deal with the different individual subjects which were investigated.

PART I. SCIENTIFIC INFORMATION CONCERNING BALLISTICS.

To secure ballistic information in France, the ordnance officers of the French army and navy were consulted in Paris and the various developments were discussed.

REPORT OF OBSERVATIONS MADE BY H. L. CURTIS
ON A TRIP TO FRANCE AND ENGLAND DURING
APRIL, MAY, AND JUNE 1919.

This trip was undertaken primarily to obtain information concerning the developments in ballistic instruments which have been made in England and France during the war. However, observations were not confined to this subject alone but a number of other subjects of interest to this Section and other Sections of the Bureau were investigated.

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To secure ballistic information in France, the ordnance officers of the French army and navy were consulted in Paris and the various developments were discussed.

Permission was then obtained to visit various proving grounds and laboratories. The proving grounds visited were those at Versailles, Gavres, Quiberon, and Bourges. Also the laboratory of the Commission des Poudres at Versailles was visited as well as a number of university laboratories in Paris where various phases of ballistic investigations were carried on.

In England many of the officers of the army, who have been working on ballistic problems, have been demobilized, but I visited a number of these at the university laboratories where they are now located. The Admiralty had been cooperating with the National Physical Laboratory in making some ballistic investigations, and I was given every opportunity to get in touch with this work. The work on intérior ballistics is being largely done at Woolwich Arsenal which I visited on two different occasions. The principal work on exterior ballistics is being done at Shoeburyness Proving Ground, which was also visited.

The following discussions will be by subject, reference being made to the places where the investigations have taken place.

1. MEASUREMENT OF THE INITIAL VELOCITY OF PROJECTILES.

The measurement of the initial velocity is the most important ballistic measurement. It has long been given first place in all proving grouna measurements. During the recent war, it was considered of such importance that

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The measurement of the initial velocity is the most important ballistic measurement. It has long been given first place in all proving ground measurements. During the recent war, it was considered of such importance that

apparatus was devised for measuring it on the battle-front. It is even suggested that the initial velocity should be taken of every projectile fired from very heavy ordnance.

(a) <u>Boulengé Chronograph</u>: At all of the proving grounds which I visited the Boulengé chronograph is the standard instrument. In France they use a modification which is known as the Boulengé-Berger chronograph. The only important modification which I could find is that there is an ammeter placed in series with the magnets and the current is always regulated to a particular value.

At a number of proving grounds there was a fall apparatus for accurately obtaining a time interval of a tenth of a second. This was used occasionally to determine the accuracy of the chronograph. At the proving ground near Versailles, the screens are so placed that the time interval is always approximately 0.1 second for each shot.

At most of the proving grounds two chronographs were used but at Bourges a single chronograph is used for determining the velocity. It is here that they claim the highest accuracy for the Boulengé. At Quiberon two chronographs are used and their accuracy is tested by interconnecting the circuits in such a way that a single key will allow both sets of magnets to be interrupted at the same instant. In this way they check the instruments against each other.

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So far as I could learn no satisfactory experiments have been made to show the limits of accuracy which can be obtained by the use of the Boulengé chronograph, though at some of the proving grounds very high accuracy is claimed. While the Boulengé is the one instrument that is used at all proving grounds, yet practically every proving ground has some auxiliary method of measuring initial velocity. These will be discussed below.

(b) <u>Schultz Chronograph:</u> At a number of proving grounds I found that they have Schultz chronographs but in no case did I find that it was being used.

(c) <u>Joly Chronograph:</u> The Joly chronograph was developed by Captain Joly of the French Artillery in order that he might obtain the velocity of projectiles while operating his battery at the front. For this purpose it proved quite satisfactory. It was also used in training areas. It is now being used at the Quiberon Proving Ground to a very considerable extent. They find that it gives an accuracy which is very nearly, if not quite, equal to that of the Boulengé. At other French proving grounds, however, it is considered to be quite inaccurate.

The principal of the Joly chronograph has been described by a number of American officers; also some of the French pamphlets describing its method of use have been translated into English. The most distinctive feature concerning it is the use of the bore wave to operate interrupters which record the passage of the shell. The

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recording instrument is a smoked drum, the speed of which is determined by the trace of a tuning fork. The greatest source of inaccuracy in the instrument is probably in the locating of the interrupters at the same distance from the line of fire of the projectile. M. Joly claims that an accuracy of one part in 300 can be obtained by the instrument. This seems justified by the results obtained at Quiberon. In my opinion it was discarded at other proving grounds without being given a satisfactory trial. In general it is not convenient for proving ground work.

(d) Chronograph for High Angle Fire: At Bourges and at Quiberon preparations are being made for obtaining velocity at very high angle fire. At Bourges two reinforced concrete towers are being built which will permit of the measurement of velocities at an angle of elevation of 30° using screens and a Boulengé chronograph. These towers appeared to me to be about 50 meters apart. To obtain the higher angles of fire they propose placing Joly interrupters on the tops of these towers and connecting them directly to the magnets of a Boulengé chronograph. It will be necessary to correct the results for the fact that the two interrupters are not parallel to the line of fire.

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to other proving grounds when necessary. Each tower consists of two upright standards which are bolted together near the top. The lower part of each standard ends in a hemispherical ball which rests in the hemispherical cup of a cement pier. There is a layer of sand between the two cement surfaces. The towers are about 50 meters apart. They are placed on a low hill, the gun position being in the valley.

(e) The apparatus Rougé is being developed at the Bourges proving ground for the purpose of getting relative velocities of projectiles from trench mortars. They did not consider that it would be of any use at all where the projectile moves with high velocity.

This apparatus consists of a carefully pivoted mirror so arranged that on the breaking of an electric circuit it is given a definite angular velocity. A wire is stretched across the path of the projectile which serves to break the electric circuit and start the mirror of the Rougé apparatus in motion. The apparatus is placed in such a position that when the mirror is in motion the image of the shell in the mirror can be seen in a small telescope whose axis is perpendicular to the axis of the mirror. If the mirror has the proper angular velocity the image of the shell in the mirror will remain stationary and this can be readily observed through the telescope. If the mirror is moving slightly too fast or too slow, the image will move slowly across the field of view of the telescope.

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In use, the speed of rotation of the mirror is adjusted until, with the expected velocity of the projectile, the image remains stationary. To determine an unknown velocity one merely observes the rate at which the projectile appears to move across the field of the telescope and the direction in which it moves. From these observations, the speed of the projectile is determined.

This instrument does not give accurate results and probably its only field of usefulness is measuring the initial velocity of projectiles from trench mortars.

(f) Cotton's Method of Measuring Velocity of Projectiles: Professor Cotton of the Ecole Normale Superieure is developing an instrument for measuring the velocity of projectiles. It will permit the reading of the velocity on an instrument in the same way that the voltage of a circuit is read on a voltmeter. I did not learn the relationship between Professor Cotton and the French army and navy.

The apparatus consists of two interrupters similar in principle but very different in design from those used by Joly in his chronograph. These interrupters are placed in the two arms of a balanced Wheatstone bridge which uses a fluxmeter as an indicating instrument. The fluxmeter, which has a scale similar to a voltmeter, is set at zero before the gun is fired; the pointer of the fluxmeter moves only when the shell is between the two interrupters. As soon as the shell has passed the second interrupter, the needle of the fluxmeter becomes stationary. If the scale

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is properly graduated, the velocity is read directly from the instrument in the same manner as the voltage from a voltmeter.

One advantage of this instrument is that the interrupters may be placed comparatively close together. Professor Cotton estimates that a distance of ten meters between the interrupters is ample. The following description was prepared soon after my visit to Professor Cotton's laboratory:

There are two "rupteurs" placed about 10 m. apart which are in the two arms of a Wheatstone bridge. (A diagram showing the method of connection is given in Fig. 1, page 10). They are placed near and both at the same distance from the path of the projectile. The bow wave of the projectile breaks the circuit at each of the rupteurs. A current flows through the fluxmeter only when one is broken and the other still closed. The deflection of the fluxmeter is proportional to the quantity of electricity which flows through it. Hence, if the current which flows when one rupteur is broken is known, the time can be computed from the deflection of the fluxmeter and its known constant. In fact, if the current is kept constant, the fluxmeter can be graduated to read directly in time, or if the rupteurs are always placed the same distance apart, the fluxmeter scale can be graduated to read the velocity of the shell directly.

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thinks that the instrument is entirely satisfactory.

He has constructed a special type of rupteur. (A diagram of this is given in Fig. 2, page 10). An iron and a silver wire are welded into the form of a cross. The silver wire serves to support the iron wire. The position of the iron wire can be adjusted by turning the Tortion head to which one end of the silver wire is attached. This is adjusted till the platinum tip of the iron wire is about 1 mm. from a gold plate on the diaphragm. When the short-circuiting key is closed, the bridge current flows through the magnet, which then attracts the iron wire, thus closing the contact on the diaphragm. When the key is opened, the current flows through the contact and the circuit remains closed. If now an intense sound wave strikes the diaphragm, the contact is broken and remains broken since the electromagnet no longer attracts the iron wire. Evidently there is a proper relationship between the periods of diaphragm and of the iron wire system, but I did not understand that Professon Cotton has as yet attempted to determine this relationship.

This complete set-up is shown in Fig. 3, page 11. It requires three wires to each of the rupteurs. Both of the rupteurs are reset by the closing of a single contact key. As the direction of deflection of the fluxmeter is always the same, the zero of the instrument may be placed at one side so that the full 90° of deflection may be used.

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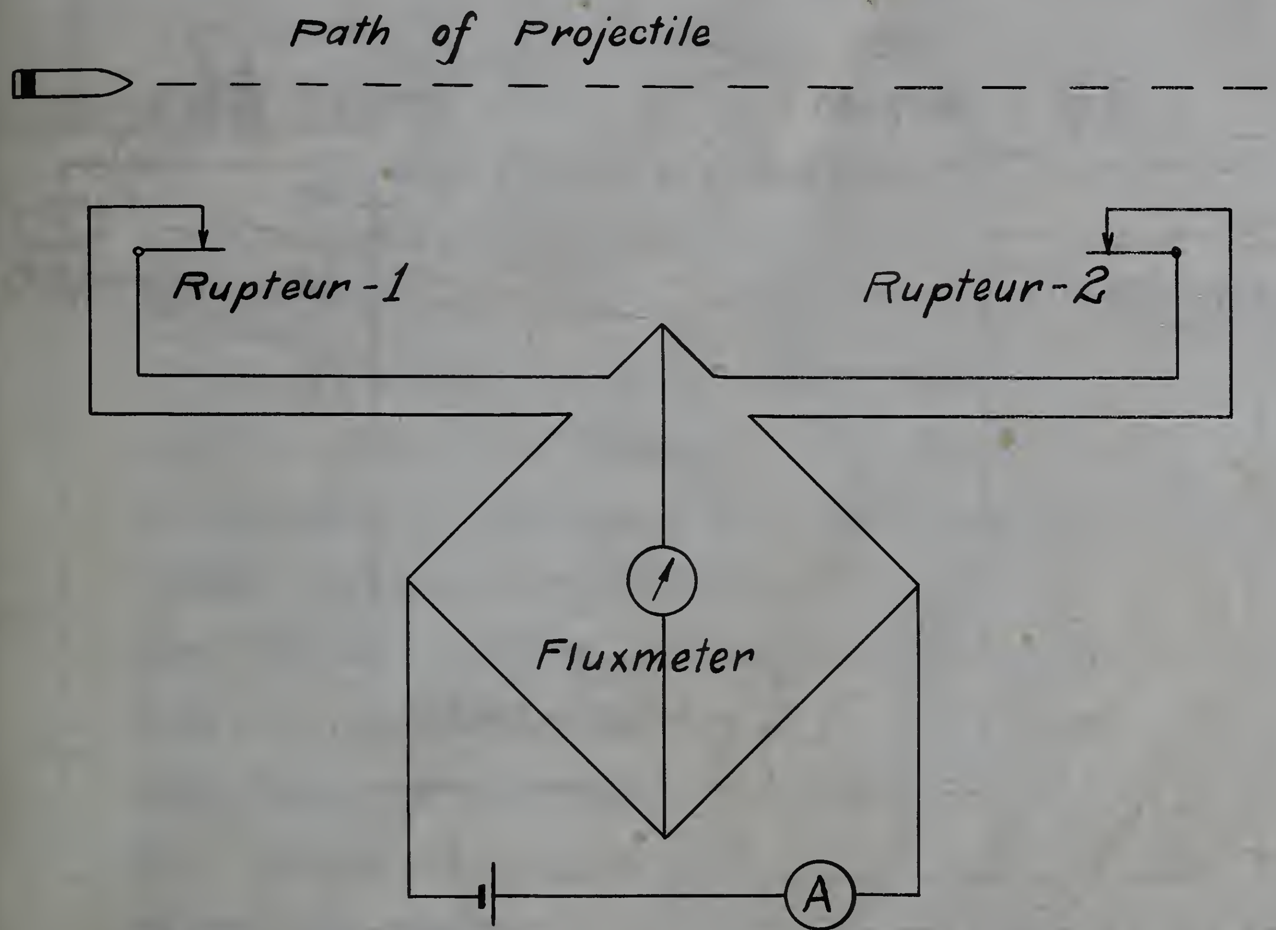

Fig 1. — Diagram to show the use of a Wheatstone Bridge with a Fluxmeter to measure Time Intervals.

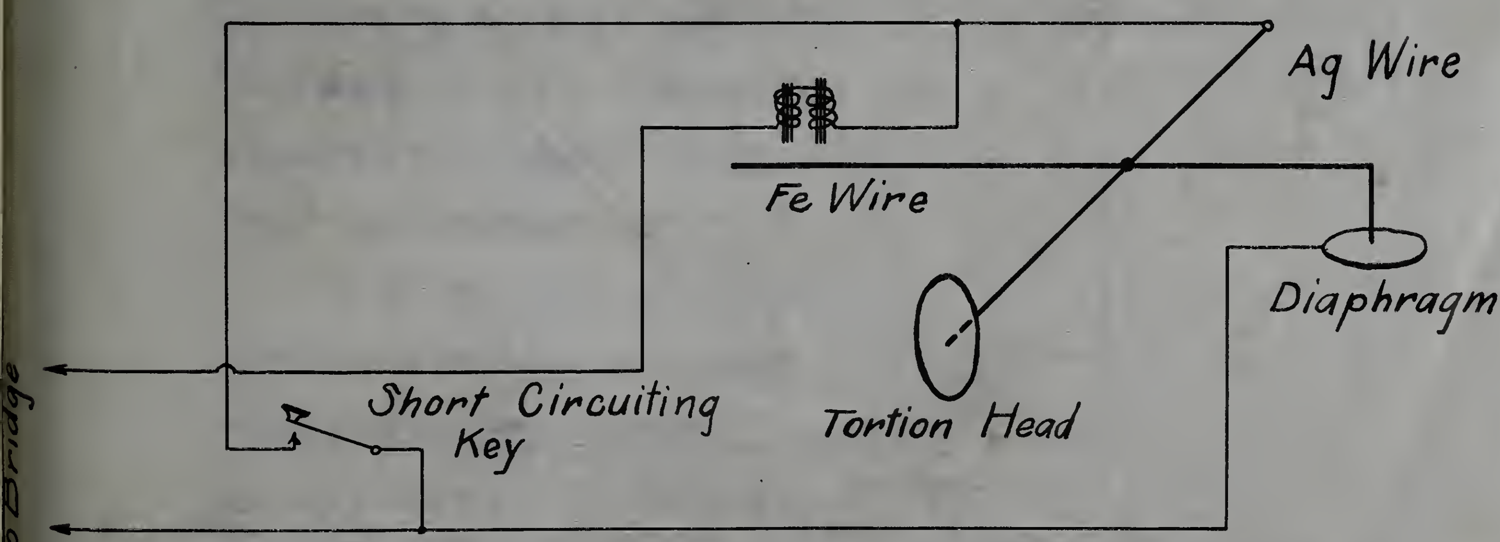

Fig 2 — Diagram of a Rupteur.

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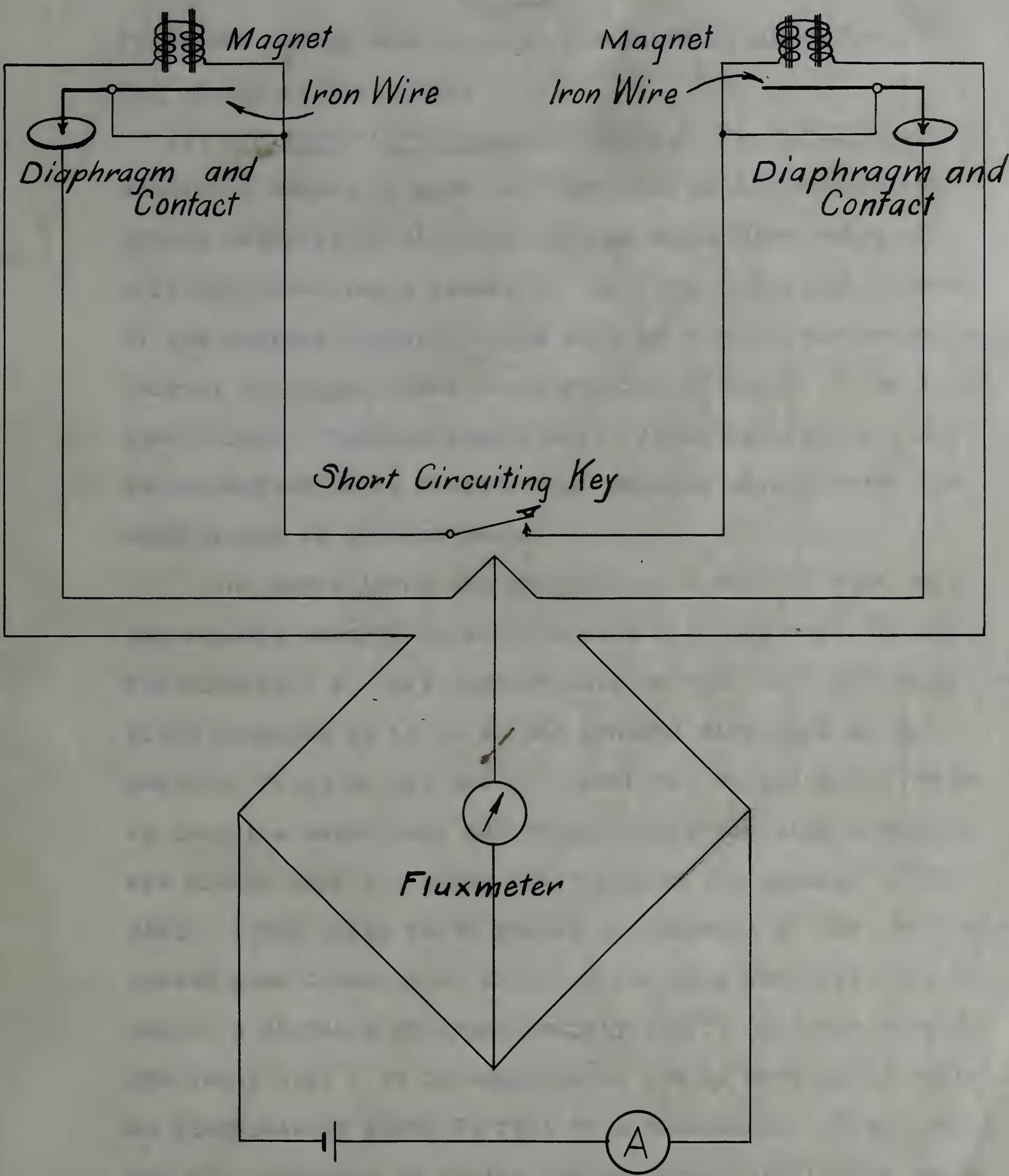

Fig 3. — Diagram showing the Set-up for measuring the Velocity of a Projectile by a Fluxmeter.

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DIVISION 1 - SECTION 2
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Professor Cotton also employs a magnifying glass for reading tenths of divisions.

(g) Velocity by Inductance Methods: In England a considerable amount of work has been done on the measurement of the velocity by shooting through magnetized coils of wire and obtaining a record of the time of passage by means of the current induced in the coil of wire. This method was largely developed under the direction of Mr. F. E. Smith of the National Physical Laboratory, Investigations are now being carried on at Shoeburyness proving ground under the supervision of Colonel Paul.

For magnetizing the projectile, a coil of wire carrying a heavy current is wound around the muzzle of the gun. The direction of this current must be such that the magnetic field produced by it is in the general direction of the magnetic field of the earth. However, as the shell tends to lose its magnetism, additional coils carrying a current are placed near the coils which record the passage of the shell. The coils which record the passage of the shell are placed some distance in front of the gun, the first one being at a distance of approximately 100 ft. and the second 150 feet, etc. It is expected to use as many as 20 coils at distances of about 50 feet from each other. These coils are all connected in series and are connected to the center string of a string galvanometer which is arranged to photograph the movements of three strings.

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current in the coil, first in one direction, then in the other, giving a characteristic record on the film. The record from each of the coils is identical. To determine the velocity of the projectile, it is only necessary to measure on the film the time from any definite point on one record to the same point on another record and to know the distance between the two coils which produced these records.

To measure the time interval on a film, a 500 cycle alternating current is sent through the two outside strings connected in series. The 500 cycle current is obtained from a tuning fork which is driven by means of a vacuum tube of the audion type. By projecting the record on a ruled screen and varying the magnification until the ten vibrations of the fork correspond with ten rulings on the screen, it is possible to obtain the time of the passage of the shell within .00005 second, always, however, on the assumption that the magnetic conditions do not vary from screen to screen.

This method is being developed with the idea that it will be possible to measure the velocity with sufficient accuracy to obtain a good value of the resistance of the air. Development work is continuing. Much remains to be done before the method can be considered as having definitely proved its worth.

(h) Retardation of a Projectile by Armor Plate: The French navy is preparing to measure the negative

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(h) <u>Retardation of a Projectile by Armor Plate:</u> The French navy is preparing to measure the negative

acceleration of a projectile when it pierces an armor plate. No definite method has been decided upon.

At Shoeburyness, Colonel Paul states that he has measured the velocity of a projectile before and after it passed through a plate of armor. I did not learn the results obtained.

(i) <u>General Conclusions</u>:

1. There is need for an instrument which can be used in the field and which will give an accuracy of at least 1%. In my opinion, the Joly chronograph, which was designed for this work, is not as suitable as the method which is being developed by Professor Cotton. Development work along the line of Professor Cotton's method seems to be highly desirable.

2. The Boulengé chronograph is the most satisfactory chronograph which is at present in use in Europe for the ordinary routine tests of a proving ground. While expensive and difficult to install, it gives a fair degree of accuracy and is in general dependable.

3. The Aberdeen chronograph is not known at any of the proving grounds of Europe.

4. There is need of apparatus for measuring the velocity at high angle fire. At Bourges and Quiberon they are building very tall towers for this purpose. Also the apparatus Rouge is being developed for this kind of work but it is only applicable to low velocity projectiles.

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ty of projectiles on shipboard. They have tried two methods both of which are unsatisfactory.

6. There is a great variety of opinion as to the accuracy of different methods. There is very little useful data which shows their comparative accuracy. It would be highly desirable to have some experiments made whereby the accuracy of the different methods can be definitely tested.

2. MEASUREMENT OF THE PRESSURE IN A CANNON

The two laboratories which are particularly interested in the measurement of pressure in a cannon are the Laboratoire de la Commission de Poudre at Versailles and the Laboratory of Internal Ballistics at Woolwich Arsenal. At other proving grounds measurement of pressure is very largely confined to the use of the crusher gage.

At both of the above laboratories, the time-pressure curve is generally measured in a bomb though both have developed gages for measuring it in a cannon. At Gavre some measurements have been made of time-pressure curves but these are not featured as a part of their experimental work.

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opinion gives considerable information concerning the quality of powders which are being tested. They also have a crusher gage suitable for putting in a cannon in which a small vibrating reed records the rate at which the copper is crushed. From this a curve of the development of pressure in the cannon can be plotted. However, this records the pressure only until it has reached its maximum and gives such a small record that accurate results are impossible. Similar instruments have been reached at Woolwich, and one has been obtained for use by the Bureau of Standards.

(b) Recording Spring Gage: Instruments have been constructed both in England and in France which record the pressure in the gun by means of the compression of the spring. The time intervals are recorded by means of a vibrating reed placed inside the instrument. The record obtained is very small and is very difficult to interpret. They are not considered satisfactory instruments.

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methods of measuring the pressure are being used:

1. The compression of a copper disc: The compression of this disc operates a mirror so that the rate of compression is registered on a photographic film.

2. Compression of lead disc: This works in the same manner as the copper disc and is photographed on the same film.

3. Compression of a spring: The compression of a spring operates a mirror which again registers on the same photographic film as the others. This spring is a solid piece of steel so that the compression is small and the period very high. With such a spring, it is unnecessary to use a damped system.

4. The maximum pressure is measured by the raising of a weight.

(d) <u>The Piezo-Electric Method of Measuring Pressure</u>:

In England a method for measuring pressure by the use of the piezo-electric phenomenon has been developed under the direction of Sir J. J. Thompson. The method was carefully outlined in an address given by Sir Thompson before the Royal Institution. An abstract of this address was published in Engineering for April 25, 1919.

When a force is applied to the proper axis of certain crystals, such as quartz, tourmaline, and a few others, a positive electric charge is developed on certain parts of the surface of the crystal and a negative charge on the other parts. If metallic plates are placed on those parts

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of the surface of the crystal where the charges are developed, they may be used to actuate an electrometer or other instrument suitable for registering electrostatic charges.

In Professor Thompson's instrument, the plates of the crystal are connected to the two plates of a Braun tube. This gives a deflection of the electronic stream in the tube. Hence, the recording system is practically without mass so that it records with great exactness the change in e.m.f. developed by the crystal. Up to the present there is not to my knowledge any data to show how much the e.m.f. of the crystal lags behind the applied pressure. It is probably very small but may have to be considered if the method is used for high explosives.

This method has been applied to the measurement of pressure in water caused by the explosion of a mine. While successful for this purpose, only very moderate accuracy has been obtained. At Woolwich they expect to try the method in a bomb, but realize that the effect of temperature will complicate the problem. It will be six months or a year before their apparatus is ready.

In my judgment, the usefulness of the method is in the measurement of the rate of rise of pressure produced by high explosives. For such work, I do not know of any other method that is applicable. For the measurement of pressure developed by propellants it seems to me that other methods are more suitable.

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(e) General Conclusions: There are no satisfactory methods of measuring the time-pressure curve in the gun. Need of this is felt in all ballistic laboratories. Active development work is being carried on at Woolwich Arsenal, and it is highly probable that they will develop a method in the course of the next few years.

3. MEASUREMENT OF RECOIL

The Siebert velocimeter is generally used for making measurements of the recoil of guns. This, however, is not considered very satisfactory and in none of the laboratories did I find that they were making regular measurements of recoil. Also, very little work has been done to show the amount of recoil at the time of the ejection of the shell.

At Woolwich Arsenal the question of the recoil of guns was being actively studied. They have been taking moving pictures of the gun at the time of firing for the purpose of determining the rate of recoil. A scale is painted on the side of the gun and some reference mark is set up just in front of this scale. Moving pictures at the rate of 300 per second are taken at the time of the firing of the gun. For determining the time at which each picture is taken, there is placed in the field of the camera a dial graduated into 100 parts in front of which rotates a hand which makes a complete revolution every one-tenth second. Each photograph shows the amount of recoil and the time at which the picture was taken. It also shows the amount of recoil at the time of ejection of the shell, provided, of

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They were very much interested in the work which we have done on recoil. At my last visit they stated that they were contemplating the construction of an instrument along the lines which we have used.

I did not at any place see in operation the method used in Italy and described by Major Veblen. This is also a photographic method but is different from the methods used at Woolwich.

In France they have done some theoretical work in an attempt to determine the velocity of the projectile by the velocity of recoil, and also to determine the pressure in the gun from the acceleration of the recoil. However, they could only make theory agree with measurement by introducing an arbitrary constant in their equation. This of course simply means that the theory has not been developed to a sufficiently high dgree so that it can be used to explain facts.

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an actual picture of the shell in flight, and, second, to get the velocity of a shell by photographic methods.

To obtain a picture of a shell, the plate or film is caused to move with such a velocity that the image of the shell remains stationary. Very clear photographs of shells as much as 155 mm. in diameter have thus been obtained. Some of these pictures were secured, as well as a complete report of the method.

To get the velocity of a shell, a plate or film is made to move at a known velocity at right angles to the motion of the shell. This gives a diagonal trace on the plate. From the known speed of the plate and the angle which the trace makes with the direction of motion of the plate, the velocity of the shell can be determined. This is not an exceedingly accurate method, but it does give some information which can not readily be obtained by other methods. For instance, it is possible to obtain the velocity of the gases which are ejected ahead of the shell, and also to determine by what interval of time they precede the shell. They also obtain the velocity of the gases in the blast and can determine the position of the shell when the secondary explosion of the powder gas occurs.

The methods are by no means perfect. They are continuing work actively under the direction of Mr. Fériet, who was formerly a professor of physics in one of the French Universities.

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in flight has been developed by Mr. L. Bull at the Institute Marey, Paris. The method used can best be understood by referring to the blue print which is attached.

At the focus of a parabolic mirror is placed a spark gap using zinc electrodes. Sparks are made to pass at regular intervals in the following manner: A large condenser having a capacity of 3/19 microfarad is charged to a potential of about 12,000 volts by means of an induction coil and a kenetron. This large condenser feeds a small condenser of about .002 microfarad through an adjustable high resistance water rheostat. The electrodes in front of the mirror are connected directly to the small condenser. The operation is then as follows:

After the large condenser has been charged, the inductance coil is cut off. The small condenser is thrown into the circuit just before the photograph is taken. It charges through a high resistance until the potential becomes sufficient to make a spark between the zinc electrodes. This discharges the small condenser and an air jet prevents any possibility of arcing. The small condenser then charges again through the high resistance and again a spark takes place. This occurs with great regularity and can be varied at will by changing the value of the high resistance. The spark is used as a source of light for taking a shadow picture of the projectile.

The parabolic mirror throws a beam of parallel light across the region in front of the gun. Through this light

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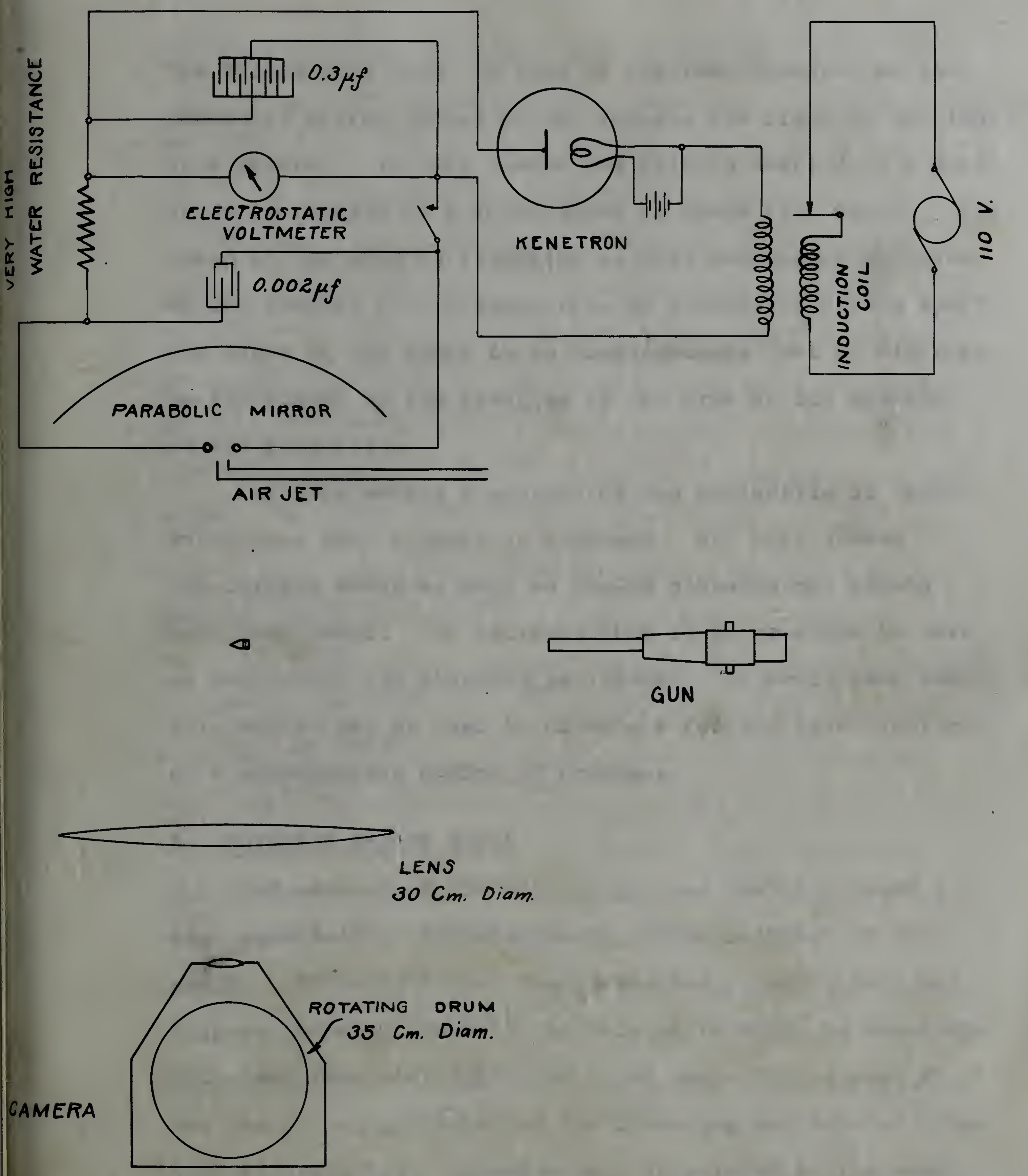

[B]UREAU OF STANDARDS WASHINGTON - D.C.	Dr. CWC	JUL. 29, 1919	M[r] BULL'S APPARATUS for PROJECTILE PHOTOGRAPHY
	Tr. CWC	Scale~None	
[D]IVISION - 1 SECTION - 2	Chkd.		
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the bullet must pass. A lens of the same diameter as the parabolic mirror (about 30 cm) focuses the light on the lens of a camera. In this camera the film is carried on a drum, which is rotated at a known speed by means of a motor. The speed of the drum is regulated so that successive pictures do not overlap with a given rate of production of the spark. The light of the spark is so instantaneous that no blurring is introduced by the rotation of the drum or the movement of the projectile.

By this method a picture of the projectile is taken every time that a spark is produced. Mr. Bull showed photographs where as many as 15,000 pictures per second have been taken. He believes that it is possible to take as many as 80,000 pictures per second. It would seem that this method may be used to advantage for the investigation of a considerable number of problems.

5. ROTATION OF THE SHELL

At several of the proving grounds they expressed a keen interest in the measurement of the rotation of the shell. It is felt that frequently the rotating band may slip on the shell so that the rate of rotation is considerably less than that indicated. At Gavre they expect to use the photographic method for obtaining the rate of rotation of the shell. A white spot is painted on the shell and this can be observed in the photographs. I did not learn the details of the method which they propose to employ.

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6. VELOCITY OF SHELL IN THE BORE OF THE GUN

Work is in contemplation at Gavre and at Shoeburyness for determining the velocity curve of the shell within the gun. At Shoeburyness some experiments have been made on an attempt to use coils around the gun. When the shell passes through this coil, it induces a current in the coil which can be measured by a sufficiently sensitive string galvanometer. However, the recoil of the gun also changes the magnetic field through the coil and these changes may be far in excess of those produced by the discharge of the shell through the coil. The method, therefore, does not appear to be a very satisfactory one and while it has not been entirely discarded it is being held in abeyance.

At Gavre proving ground, General Bourgoin has designed a very elaborate apparatus for determining the acceleration of a shell in a gun. Reduced to its simplest elements, this may be described as follows: Inside the shell there is placed a heavy mass held in position by a calibrated spring, and so pivoted that it does not turn with the shell. When the gun is fired, the acceleration of the shell compresses the spring and the turning of the shell can be used to determine the position of the shell in the gun. Hence, a stylus attached to the mass will describe on the interior of the shell a curve from which the time acceleration curve can be determined. From this the velocity of the shell can be obtained by integration and from this the pressure inside the chamber of the gun. This apparatus has not yet

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7. VELOCITY OF A PROJECTILE OBTAINED ON SHIPBOARD

In England an attempt was being made to determine the velocity of projectiles when fired from a ship at sea. The method which they were trying to use was to get the time from the beginning of recoil until the ejection of the shell. They expected that it would be necessary to make experiments on every type of gun which they wished to use at the proving ground. It was pointed out that our work indicated that this method will not be satisfactory.

8. TIME OF EJECTION OF A SHELL

Three methods of measuring the time of ejection of the shell are in use:

1. In some places they are using the wire across the muzzle and recording on some type of chronograph. It is quite generally appreciated, however, that this is not a satisfactory method.

2. The shell is photographed at the time of its ejection. This photograph, if properly timed, will give a very satisfactory record of the ejection of the shell. However, it is very difficult to get a photograph at the proper instant.

3. An inductance method is in use at Shoeburyness. If the shell is magnetized, then when it leaves the gun it produces a change in the magnetic flux in the gun itself.

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Hence, if a coil of wire is wound around the gun, there will be a considerable induced e.m.f. in the coil at the time of the ejection of the shell, caused by this change of flux. However, it is not known at just what position of the shell this change of flux is at a maximum. It is probably about the time that the base of the shell leaves the gun, but as yet sufficient experimental work has not been done to determine this accurately.

9. MEASUREMENT OF AIR RESISTANCE

The measurement of air resistance is fundamental for ballistic computations and the need for better values is quite generally felt. This is especially true in England where two different kinds of apparatus are being devised for making these measurements.

1. The method of measuring velocity by shooting through a number of coils in series, as already described, was devised primarily for the purpose of determining the resistance of the air to the passage of the shell. As soon as the method is perfected, it is proposed to use it for this purpose to a considerable extent.

2. Another method of studying air resistance is the use of high velocity wind tunnels or air jets. In England no work has been done at present but a committee of the army and navy is seriously considering the question. It is proposed to install suitable apparatus at the University of Cambridge, placing it in charge of Mr. R. H. Fowler, a

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Mr. Saphores showed me the operation of the apparatus in some detail. A description of my observations are available to any one who are interested.

10. TIME OF FLIGHT OF PROJECTILES

Special instruments for measuring the time of flight of projectiles have been devised both at Quiberon and Bourges. The instrument used at Quiberon requires two observers. The first observer looks into a pair of ordinary binoculars. However, he sees a different object with each eye, each of these being placed close to the object glass of the binoculars. With the right eye he sees a clock having only one hand. The dial of the clock has 100 divisions. The hand of this clock makes a complete revolution in a second so that one one-hundredth of a second can be read on the clock. With the left eye the observer looks into a space which is dark except when illuminated by an electric spark. When the shell leaves the gun, it breaks the circuit which

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At Bourges they use an acoustical method for obtaining the time of flight when using fused projectiles. The projectile is fired so that it will burst a short distance above the ground. An interrupter is placed at a known distance from the point of burst. A second interrupter is placed at the same distance from the muzzle of the gun. Each of these interrupters is connected to recorders which record on a smoked tape. On the same tape a clock such as used in the French sound ranging apparatus gives the time record. The recording styluses are placed at such distances apart that if the paper is running at normal speed and the burst occurs at the expected position, the two records will be side by side.

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11. INITIAL STABILITY OF PROJECTILES

Mr. R. H. Fowler of Trinity College, Cambridge University, has done considerable mathematical work in the study of the initial stability of projectiles. He expects to prepare a paper on the subject which is to be published within a short time in some of the scientific publications of England. Mr. Fowler thinks that the initial yaw of the shell may be produced either before or after the rifling band leaves the muzzle. In the first case, due to the reaction of the gun, the center of gravity of the projectile will be given a velocity perpendicular to the line of flight. However, if the yaw is produced by the blast after the projectile leaves the muzzle, it will merely rotate the shell around the center of gravity. These two cases will require separate mathematical treatment.

12. JUMP AND WHIP OF GUNS

The only method for measuring jump and whip which is used in France or in England is by the use of so-called jump cards. A card is placed so that its center is directly in line with the axis of the gun. When the gun is fired, the shell may or may not pass through the center of the card. If it passes above the center of the card, the gun is said to have jumped. From the amount of jump and the distance of the card from the gun, the angle of jump is determined. Reports on the result of this work

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in England have been received.

13. GENERAL OBSERVATIONS OF BALLISTIC INSTRUMENTS

The behavior of primers has been studied at Greenwich observatory, but I did not see any of their results. I did not find any place where they are studying the time of ignition of the powder. In England they assume that the initial recoil coincides with the initial movement of the shell. However, they have not done any experimental work to show this is the case.

General Bourgoin, Chef de la Commission de Gavre, believes that the two important improvements which are about to take place along the line of ballistics is the use of envelope powder and the development of a shell which will use the Chilowsky process. He has in mind a powder which consists of three distinct materials arranged so that a very rapid burning powder, carrying a very large amount of energy (a powder burning more slowly and the whole surrounded by a coating of nitro-cellulose powder. The object of envelope powder is to obtain more energy in the same mass and to produce a powder in which the pressure throughout the movement of the projectile in the barrel of the cannon will be approximately constant. He believes that this powder represents as great an advance over the smokeless powder as smokeless powder was over black powder.

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that have been conducted were carried on with an idea of obtaining useful results quickly and did not attempt to get at the fundamental principles involved. There are a considerable number of variables and each one must be investigated separately before the process can be perfected.

14. SCIENTIFIC EQUIPMENT AND PERSONNEL OF PROVING GROUNDS

During the war, the scientists of France were drawn upon to supply necessary technical information to the French proving grounds. Most of these men have been demobilized but Mr. Fériet is still retained at Gavre to complete work on the photography of projectiles. Comparatively little scientific work is in progress in France at the present time.

In England, scientists from the universities very largely entered war work, and were assigned to problems for which they were suited. A large part of the ballistic investigations carried on by the army and navy were in charge of these men. However, the National Physical Laboratory was frequently consulted by the army, navy, and the air ministry. Now that peace has been declared, it has been found especially advantageous for the army and navy to have connections with the government institutions of fixed policy and standing. They therefore seem to be depending more and more on the National Physical Laboratory for scientific advice. However, this laboratory is only half the size of the Bureau of Standards, so that it is frequently necessary

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for the army or navy to undertake a large part of the work under the supervision of the National Physical Laboratory.

The physical equipment of the French proving ground is generally well adapted for the purposes to which it is applied. At Gavre where it is proposed to do long range testing, a complete aeronautical equipment is available with airplanes and captive balloons, so that all necessary ballistic corrections can be made. There is a similar complete equipment at Shoeburyness. Electric power, storage batteries, chronometers and the like are available at all places. In general it would seem that there is less difficulty in securing satisfactory equipment than in securing the personnel.

15. PRINCIPLES USED IN THE DESIGN OF SHELLS

Comparatively little new information has been evolved concerning the principles to be used in the design of shells. The French are using a stream line base for their shells, but the utility of this is doubted by the English. All are agreed that there is not sufficient scientific data to permit of a rational design of shells.

16. DECOPPERING OF GUNS

I understand that the original work on the use of a lead tin alloy for decoppering guns was done at Camp Mailly by General St. Clair de Ville. However, I have been unable to get any account of the original experiments. At present there is a great difference of opinion among officers as to

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the value of the method. Some say that if continually used it will injure the gun; others state that they know it will produce very remarkable results on a gun which has a bad coating of copper. There seems to be considerable need for further experimental work, both in the matter of determining the best material to use and in determining the frequency and method of using it. Those who are interested in this subject should consult the report by Major Anderson of the War Department.

17. METEOROLOGY

I did not attempt to make any extensive study of the methods used to obtain meteorological information. However, I did have some consultations with Colonel Blair, who was in charge of the meteorological service for the U.S. Army. I also made some inquiries at proving grounds where meteorological observations were taken.

There are two distinct properties of the air that need to be measured, viz., density and velocity. Corrections must be applied for both of these. These corrections may amount to as much as 3% or 4% in range, and as large as 10 mils in deflection.

To determine the air density, it is necessary to know the temperature, pressure and humidity (or vapor tension). If these can be determined to a reasonable height, then the behavior for greater heights can be predicted. To obtain this data it is necessary to send up a balloon or airplane

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to make the necessary observations, especially on the temperature and humidity. The pressure can be fairly well determined from measurements at the surface as the decrease in pressure with height follows a known law.

The wind correction is the important correction. Three methods of obtaining this are employed: (a) by a sounding balloon with theodolite; (b) by sound ranging methods; (c) by shell burst.

(a) A rubber balloon is filled with hydrogen until it has a certain lifting power (150 gm). It is then let free and observations made on it every minute by a theodolite. The rate of ascension is taken to be uniform. Hence from the angles, as read on the theodolite, and the height at the time of the reading of the angles, the position of the balloon at the end of every minute can be determined. From these positions the average wind in the region through which the balloon passes from minute to minute can be determined.

(b) A balloon is sent up with a bomb which has a fuse timed to explode at a definite height. On the ground are seven stations of a sound ranging system. They range on the bomb, hence determining its location. Then another balloon with bomb timed to explode 500 m higher is sent up and its position located. The drift, and hence the average wind, in that layer is determined from these measurements. As the sound ranging is in three dimensions, it is more difficult than surface sound ranging, but the theory has all

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This method is more expensive, more dangerous, and gives less satisfactory results than the sounding balloon, but it is the only one that can be used in cloudy weather. Hence its development is important, especially in cloudy areas.

(c) Observation of the drift of the smoke from anti-aircraft shells. These are fired to explode at a known height and at regular intervals. If they are above a cloud, an airplane flies along a short distance to one side of the row of smoke puffs and observes the time of passing each of the puffs. From the speed and direction of the airplane, the velocity of the wind can be determined.

The results of both wind and density measurements must be integrated over the path of the projectile. The air is divided into 500 m layers and the effect on the shell in each of these layers is considered. A method of doing this rapidly has been worked out, so that the corrections will be known within a minute after the observations are completed.

For high angle fire, the conditions in the surface layer are relatively unimportant. The shell spends the most of its time in the upper layers. Conditions in the upper air do not change so rapidly as in the lower layer and are more constant over fairly large areas. Hence, the observations of one station may be used over a large area and it is only necessary to make the observations every two or

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Instruments. No new instruments for measuring temperature, pressure, or humidity, have been developed. There is need of improvements in these.

Colonel Blair will doubtless make a complete report on the work of the meteorological section of the army and this should be consulted by those who are interested in this work.

At proving grounds, it is necessary to be able to correlate work from day to day. In high angle fire, this is possible only if complete meteorological data is available.

18. PUBLICATIONS OF BALLISTICS

During the war no important books on ballistics appeared. Most of all important experimental work was written up in the form of confidential documents. Following the war, General Sugot prepared a five volume work on ballistics, which is used for instruction in the French schools. This work was published as a confidential document and can only be obtained through the military authorities.

General Carbonnier is preparing a new edition of his book on ballistics which will be published for general distribution. The manuscript is practically completed and should be published within a year.

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A number of English scientists are preparing articles on special problems which will appear in the near future. These will be published in the Transactions of the Royal Society, the Proceedings of the Royal Society, or in other scientific publications. It is my opinion that the French will be somewhat slower than the English about publishing results of scientific investigations on ballistics.

18. SOUND RANGING

Four different uses for locating objects by means of sound waves were used during the war. Guns were very successfully located by means of a muzzle wave which is set up at the time of firing the gun. Airplanes were located by sound ranging methods, and sound ranging methods were used to determine the velocity of the wind at high altitudes. Mining operations were also located by means of sound waves through the earth. While these methods were not studied in detail, some observations were made on each of these subjects.

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The French used a number of different systems. On only one of these did I get any information. The Cotton-Weiss method of sound ranging has been considerably improved since the first designs were brought to America by the French Scientific Commission in 1917. Professor Cotton showed me the latest type of sound ranging instrument. The recording device is the same as he has always used but the instrument for detecting the sound wave is quite different. It is merely a small diaphragm having a contact on one surface. This contact is connected through a series of electromagnets in such a way that as soon as the circuit is opened the contact is drawn a considerable distance from the diaphragm. When a sound wave strikes the diaphragm it causes it to vibrate and opens the circuit. This is registered by means of a fluxmeter in the same way as the original instrument.

(b) <u>Location of Airplanes by Sound</u>:

Professor Perrin devised an apparatus for the location of airplanes by sound. The sound is received on a set of cells and conveyed to the ear of the observer by a tube. There are four of these sets of cells arranged in the corners of a square. The pair at the ends of the diagonal are connected to the two ears of an observer who can rotate this pair about an axis through their diagonal. This observer keeps the apparatus rotated so that the sound from the two

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sets of cells reach him in the same phase (sound directly ahead). The other observer does the same, so that a perpendicular to the plane of the cells passes through the airplane. The apparatus is made registering so that the path of the plane can be determined.

(c) <u>Determination of the Velocity of Wind at High Altitudes</u>.

This method has been briefly described under meteorology. It consists in locating the point of an explosion which takes place at some distance above the ground. The French use seven stations for this work. I have never seen the theory but understand that it has been carefully worked out

(d) <u>Subterranean Sound Detection</u>.

I visited the laboratory of Professor Perrin where the original investigation on geophones was carried out. They had made the same investigations that were made at the Bureau of Standards and in general arrived at the same conclusions.

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PART II. INVESTIGATIONS OF THE SUB- MARINE MINES

At the request of Admiral Strauss in command of a mine sweeping force I visited the mine sweeping fleet which is based at Kirkwell, Orkney Islands. The particular problem on which I was asked to give advice was concerning the explosion of a mine which came in contact with the sweep wire. A device which protects a ship against the mines which have been planted in the North Sea has been designed by Lieutenant Nichols, and is working satisfactorily. This device consists in making the ship of positive potential so that when the mine strikes the ship a current is sent through the firing device of the mine in the opposite direction from that which is produced by the sea battery. As a polarized relay is used in the mine, the mine will not fire if the current is sufficiently large.

To produce the necessary positive potential on the ship, the hull of the ship is directly connected to the positive pole of a generator capable of developing at least 200 amperes. The negative pole of the generator is connected through a resistance to the insulated cable of No. 0 copper wire. This cable is about 600 feet long and is bare for a distance of 10 to 12 feet. It is allowed to trail behind the vessel with the bare portion at the end farthest from the vessel. When 150 amperes flows in the protective wire, the voltage drop in the protective wire, and the sea water return, is about 20 volts. The remainder of the potential

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must be taken care of by the regulating resistance.

Experiments show that the arrangement described above protects the ship; that is, if the antenna of a mine comes in contact with the ship, the mine will not explode. It was the desire of Admiral Strauss to extend this method so that the kite wire, kite, and if possible the sweep wire, would also be protected. If a steel wire is connected directly to the ship so that it is the same potential as the ship, it was found that by increasing the current in the protedtive device by about 20% the ship would have the same protection as in the first place. However, measurements of potential along the sweep wire show that the potential dropped very rapidly after the wire entered the water so that within a comparatively few feet the potential was so low that if the antenna of the mine came in contact with the wire, the mine would be fixed. This at first seemed very difficult to explain since it is possible to protect a large vessel having a very large exposed surface yet is unable to protect a sweep wire having a very small surface. It is explainable when it is considered that when dealing with bodies immersed in a conducting medium the amount of current which leaves an object depends not only on the surface but also on the shape of the surface. The comparatively small radius of sweep wire permits a large amount of current to be conducted from the wire into the water, hence producing a rapid drop of potential along the wire.

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Since it was not possible to protect a bare wire electrically, consideration was given to other methods which might protect the kite wire. If the kite wire is insulated, small breaks in the insulation will not be dangerous, since small portions of the wire may be protected by means of an electrical protecting device. The insulated kite wire which was on hand in the fleet was not satisfactory since it crushed badly on being reeled in. Some experiments were made to determine whether it would be possible to insulate a wire by the use of canvas and an insulating varnish. It is believed that this is possible but further experiments will be necessary.

If a manilla hawser is used for the kite wire, there is no danger of the mine exploding if it comes in contact with this hawser. There have been some difficulties in that the hawser is likely to be cut by the antenna of the mine. However, this difficulty did not seem to be as serious as the difficulties which resulted from an attempt to use an insulated kite wire. Hence, it was recommended that the hawser continue to be used for the kite wire until a satisfactory cable has been developed.

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